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# Complementary Limb Motion Estimation based on Interjoint Coordination using Principal Components Analysis

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*Abstract*— **For the restitution of walking of a hemiplegic patient by means of a motorized orthosis, as well as in intelligent prosthetics, a major challenge is the coordination of healthy and robotically assisted limbs. The approach suggested here employs the method of Principal Components Analysis to first analyze the coupling of human Degrees of Freedom (DoFs) in healthy subjects. Based on this knowledge, adequate motion for inoperable DoFs in impaired patients is estimated on-line from sound limb motion. Thus, the intention of a partially paralyzed person or an amputee can be deduced from residual body motion, in order to coordinately actuate or supervise the impaired limbs. To evaluate the approach, simulations with recorded gait trajectories of healthy subjects are performed. The results of these theoretical investigations show a promising potential.**

## I. Introduction

A stroke freqently affects motor regions in the brain in a way that only one half of the body is impaired. By means of an actuated orthosis, a so-called exoskeleton, the necessary joint moments can be replaced, in order to enable the patient to walk. The challenge now lies in the coordination of the healthy body half with the paretic one. The patient would most probably be overwhelmed by a complete artificial control of each of the inoperable joints separately. Therefore, the underlying control should detect the patient's intention as far as possible and actuate the limbs coordinately.

The impedance-based trajectory adaption suggested by [1] enables the patient to deviate from a given physiologically correct reference trajectory within a variable margin. This way, the reference trajectory gradually adapts to the patient's individual gait pattern. However, this requires sufficiently coordinated activity in the paretic leg.

Another approach to generate reference trajectories is by observation of sound limbs, which might reveal the patient's movement intention. For example, [2] suggests to observe thorax acceleration in order to detect the intention of a paraplegic patient to stand up.

This paper proposes a general approach to the estimation of joint motion from known limb motion, given that there exist linear correlations between them. The focussed application is primarily human gait, however, the considerations are transferable to other synergistic motion patterns.

In human gait, it has been observed that joint angle trajectories show strong linear correlations, called "synergies" [3]–[5]. This observation indicates a reduced set of manipulated variables. Obviously, our brain has developed such refined control methodologies to deal with the redundancy or "abundancy" [6] of human Degrees of Freedom (A problem first referred to as "motor equivalence" by Bernstein [7]). However, although the coupling of resulting joint variables is known, the driving control variables themselves and the way how the brain generates them remain highly speculative. Some are convinced of the existence of a so-called "central pattern generator" in the human spine [8], as it can be found in animals [9]. However, the mechanisms of motion synergy generation seem to be not unalterably inborn, but adaptive, as has been shown by [6]: Their finding is that patients with partly impaired limbs still exhibit synergistic reaching, but with altered synergy patterns. This means that coupling still takes place, but the form of coupling itself has been relearned, adapted to the new constraints, which have been imposed by the lesion.

These findings about human movement coordination motivated the idea of constructive movement generation [10] and resulted in several approaches on automated gait pattern generation with a reduced subset of control variables, be it for the purpose of animation [11], [12] or robot gait [13], [14]. In [15], a robot controller with central pattern generator is described.

For the application of robotic gait rehabilitation for partially paretic patients, the above mentioned findings on joint synergies can be applied in a promising way. Knowing the form of synergies, even without any further knowledge about the underlying control variables, can help to estimate the intended motion of impaired limbs by observation of sound limb motion, and actuate the inoperable limbs accordingly. Ideally, this would lead to a cooperative motion of healthy and robotically assisted limbs, and eventually to a restitution of a normal gait pattern. As this proposed interface provides the user with a large control scope, probably some training will be necessary. However, the form of reconstruction will support and lead to a physiologically correct gait pattern.

It can be argued that a physiological trajectory might not be optimal for every patient, but should be replaced by an individual (presumably asymmetric) gait pattern with a focus on functionality. However, the described intention estimation process could cope with an asymmetric gait pattern, if a synergistic reference gait pattern was provided. This could be the walking pattern of another patient in a more advanced therapy stage, whose brain has probably developed new, altered synergistic strategies, as can be expected from the above mentioned synergy adaptation [6]. The reference can also be the result of a complex optimization process taking into account the patient's lesion. In patients where a complete

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recovery can be expected, however, the physiological gait pattern derived from healthy subjects will probably be the most suitable one for training, given that gait patterns of different subjects walking with the same normed velocity are astoundingly similar [16].

The cooperative and physiological trajectory generation can not only be used for joint actuation, but also for supervision: During rehabilitation, a correct synergistic actuation of the limbs can be supported and reinforced.

The coupling of upper and lower extremities during walking, which is even visible to the naked eye, can unfortunately not be exploited, because stroke patients need their arms to hold on to a supporting device, in order to feel more secure.

Another field of application may be found in intelligent, actuated prostheses for above-knee amputees, such as the "C-Leg" (Otto Bock HealthCare GmbH, Duderstadt). In above-knee amputees, hip motion of the ipsilateral leg is mostly preserved, so an intelligent prosthesis can estimate the intended knee joint motion based on contralateral knee and hip motion, and ipsilateral hip motion (The ankle is generally not actuated, because elastic foot prostheses allow for a good walking performance). Even if the interface might turn out to be difficult to learn, the patients have a permanent benefit once they master the control of their prosthesis. The disadvantage is that they need to wear sensors on their sound leg as well, complicating the donning and doffing and cosmetics of the prosthesis. Therefore, estimation should not rely on arm motion in this application either.

This paper contains a brief review of PCA, followed by its use for the proposed motion intention estimation. Then, simulations concerning the two example applications are described: rehabilitation of hemiplegic patients and intelligent above-knee prostheses.

## II. Data Reconstruction using Principal Components Analysis

Principal Components Analysis (PCA) [17], [18] is a general approach to data compression, where statistical (linear) correlation is exploitet. If $n$ measurements of the $d$-tuple $\boldsymbol{x}^T = (x_1 ... x_d)$ are given, and if linear correlations between the composing variables $x_i$ exist, then data reduction can be performed by choosing $p < d$ appropriate linear combinations of the variables $x_i$. PCA calculates these optimal linear combinations through a rotation of the coordinate system. In order to perform PCA, the $x_i$ data need to have zero mean, possibly requiring prior subtraction of the mean value. Frequently, it is also advisable to norm the data to a standard deviation of 1, especially when dealing with strongly varying magnitudes among variables. The (symmetric) covariance matrix $\mathbf{M}$ includes information about the variance of each variable, as well as their correlations. Its components are approximated from measured data as follows:

$$M_{ij} = \frac{\sum_{k=1}^{n} (x_{i,k} x_{j,k})}{n-1}. \tag{1}$$

It can be shown that the $d$ eigenvectors of $\mathbf{M}$ form the orthonormal basis of the optimal coordinate transformation: The matrix $\boldsymbol{\Gamma}$ formed by the eigenvectors, which are sorted in descending order of the corresponding eigenvalue, maps $\boldsymbol{x}$ on the new coordinates $\boldsymbol{y}$ with

$$\boldsymbol{y} = \boldsymbol{\Gamma}^T \boldsymbol{x} \ . \tag{2}$$

The first component $y_1$ has the maximum variance that can be reached by projecting $\boldsymbol{x}$ on an arbitrary unit vector. Recursively, this is valid for the remaining $y_i$, such that the last component of $\boldsymbol{y}$ has smallest variance.

Due to the fact that for orthonormal matrices, the inverse is equal to the transpose, $\boldsymbol{x}$ can be reconstructed from $\boldsymbol{y}$ by

$$\boldsymbol{x} = \boldsymbol{\Gamma} \boldsymbol{y} \ . \tag{3}$$

Neglecting the last components of $\boldsymbol{y}$ (which is equivalent to omitting the corresponding eigenvectors in $\boldsymbol{\Gamma}$ and $\boldsymbol{\Gamma}^T$, respectively), the data is reduced in a way that the least information is lost. A reconstruction of the $x_i$ from the lower-dimensional coordinates $y_i$ with $\boldsymbol{y} \in \mathbb{R}^p, p < d$, leads to a least-squares optimal fit of the original data.

The redundancy in the data, which is revealed by the analysis of correlations, can not only be used for compression, but also for reconstruction of incomplete measurements. The equation system (3) is overdetermined, if the number $d - q$ of known components of $\boldsymbol{x}$ is at least equal to the dimensionality $p$ of $\boldsymbol{y}$. Thus, it can be solved for unknown $y_i$ and partly unknown $x_i$. Assuming that the first components $\boldsymbol{x}_1 \in \mathbb{R}^{(d-q)}$ of $\boldsymbol{x}$ are known, and the remaining part $\boldsymbol{x}_2 \in \mathbb{R}^q$ is unknown, (3) is separated into

$$\boldsymbol{x}_1 = \boldsymbol{\Gamma}_1 \boldsymbol{y} \ , \quad \boldsymbol{x}_2 = \boldsymbol{\Gamma}_2 \boldsymbol{y} \ , \tag{4}$$

with $\boldsymbol{\Gamma}_1 \in \mathbb{R}^{(d-q) \times p}$ and $\boldsymbol{\Gamma}_2 \in \mathbb{R}^{q \times p}$ being the corresponding submatrices of $\boldsymbol{\Gamma}$. Thus, $\boldsymbol{x}_2$ is reconstructed from $\boldsymbol{x}_1$ by

$$\boldsymbol{x}_2 = \boldsymbol{\Gamma}_2 \boldsymbol{\Gamma}_1^{\Box} \boldsymbol{x}_1 \ , \tag{5}$$

with $\boldsymbol{\Gamma}_1^{\Box}$ being the left pseudoinverse of $\boldsymbol{\Gamma}_1$:

$$\boldsymbol{\Gamma}_1^{\Box} = (\boldsymbol{\Gamma}_1^T \boldsymbol{\Gamma}_1)^{-1} \boldsymbol{\Gamma}_1^T \ . \tag{6}$$

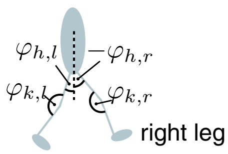
The number of eigenvectors used in $\boldsymbol{\Gamma}$ can be chosen based on the cumulative fraction of their eigenvalues of the sum of all eigenvalues. This percentage gives an estimate of how much information will be preserved during compression.

For example, if $x_1$ and $x_2$ are scalar, the first principal component stores the direction of a straight line fitting all $n$ data pairs $(x_{1,k}, x_{2,k}), k = 1...n$. The precision of fit of this line is expressed by its eigenvalue divided by the sum of eigenvalues. A value near one indicates strong linear dependence.

## III. Motion Intention Estimation

As described in the previous section, PCA provides a method to reconstruct missing values in incomplete data sets, given the condition that there is a linear correlation between known and missing data. For the application of gait restitution for partially paretic patients (or amputees),

”known data” is the motion of healthy limbs, ”missing data” is the motion of impaired (or missing) limbs, and correlation information refers to interjoint coupling during walking motion.




$$\boldsymbol{\varphi} = \begin{pmatrix} \varphi_{h,r} \\ \varphi_{k,r} \\ \varphi_{h,l} \\ \varphi_{k,l} \end{pmatrix}$$

Fig. 1. Joint angle conventions used in this paper.

The vector $\boldsymbol{x}$ now contains current joint angles and (optionally) their first and second time derivatives

$$\boldsymbol{x}^T = ((\boldsymbol{\varphi}^T)_n, (\dot{\boldsymbol{\varphi}}^T)_n, (\ddot{\boldsymbol{\varphi}}^T)_n) \ . \tag{7}$$

The index $n$ denotes that all variables are normed to have zero mean and standard deviation 1 (velocities and accelerations are inherently mean-free). For a first simple approach to human gait, the knee joint angle $\varphi_k$ and the hip joint angle $\varphi_h$ of both legs in the sagittal plane are included in the analysis, according to the conventions of Fig. 1.

Prior to intention estimation, joint synergies need to be analyzed using recorded trajectories for all DoFs during gait. This analysis is performed using PCA, as described in the preceding section. The result is the coupling matrix $\boldsymbol{\Gamma}$.

Once $\boldsymbol{\Gamma}$ is known, intention estimation can be applied for the on-line generation of reference motion for inoperable joints, using the instantaneous motion of sound limbs. Before, all variables are sorted into known motion data $\boldsymbol{x}_1$ (normed angles of the observed sound limbs, and normed time derivatives) and unknown motion data $\boldsymbol{x}_2$ (motion variables of inoperable limbs), as described in section II. In each time instant, $\boldsymbol{x}_2$ is reconstructed from $\boldsymbol{x}_1$ using (5). The reconstructed values need to be augmented again by mean and standard deviation to yield the estimated reference motion consisting of $\varphi_{i,e}$, $\dot{\varphi}_{i,e}$ and $\ddot{\varphi}_{i,e}, i = 1...q$.

As stated in the preceding section, if $q$ coordinates of the vector $x$ are unknown, then $\boldsymbol{\Gamma}$ may include $d - q$ principal components on the maximum. This will most probably be the case, because generally only very few principal components account for a cumulative percentage of over 90% of a synergistic movement [4] and thus describe the inter-dependencies sufficiently.

## IV. Example: Estimation of Hip and Knee

First, the example of gait rehabilitation of hemiplegics shall be considered. For this purpose, the motion of one leg (hip and knee in the sagittal plane) is estimated in dependance of the contralateral leg motion.

### A. Simulative Evaluation with Recorded Gait Patterns

The trajectories used for all simulations are taken from the Carnegie Mellon database (http://mocap.cs.cmu.edu). 10 healthy male subjects are included in the analysis, with database numbers 2, 6, 7, 8, 16, 35, 38, 39, 43, and 55. Unfortunately, there is only this comparably small number of straight line normal walking (E.g., several subjects in the database re-appear under different numbers or walk in circles). For a female group, not enough trajectories are available. The chosen subject group still is extremely heterogeneous, concerning height, age and weight.

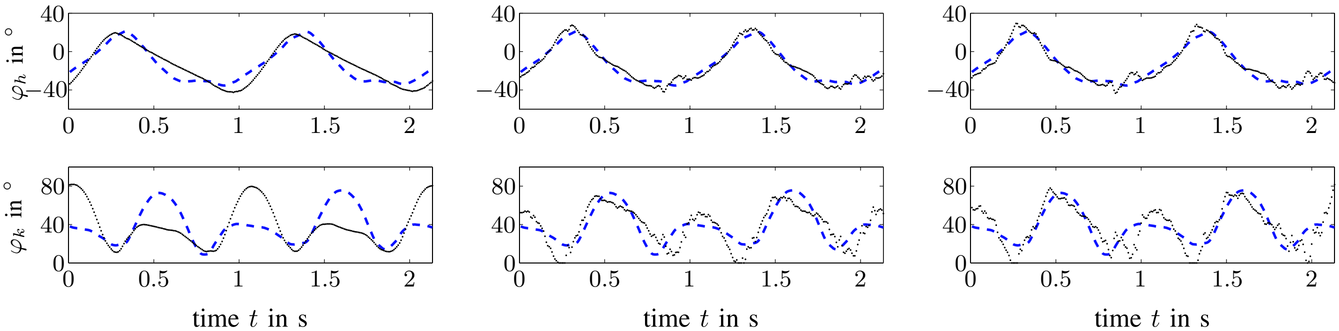
The simulative evaluation of motion intention estimation now is as follows: The recorded gait pattern of one subject is analyzed, yielding statistical characteristics for norming (mean and variance of angles, and variance of the time derivatives) and the eigenvector matrix $\boldsymbol{\Gamma}$. Then, reconstruction performance is evaluated by a simulated impairment: The corresponding motion variables are eliminated from the trajectories and subsequently reconstructed from sound limb motion using (5). Finally, the reconstructed motion is compared to the original motion. Fig. 2 shows the PCA-reconstructed motion values of subject 39. Displayed are three different reconstructions: One based on PCA with angle data only, one with angles and angular velocities, and one also including accelerations.

The number of principal components used differs in each case, because always the number yielding the best reconstruction result has been chosen. E.g., when velocities and accelerations are included, 5 PCs result optimal. This is not a large number, considering that $4$ DoFs times $3$ derivations $= 12$ variables are used in the acceleration case, which confirms again strong linear coupling. The results show that an inclusion of time derivatives improves the results, but also adds noise.

### B. Generalization: Averaged Gait as a Reference

The results of the preceding section are not practical, because the original gait pattern of the patient has most probably not been recorded. A practical way out of this problem is to use synergistic information of comparable healthy subjects. In order to evaluate the inter-subject similarity concerning joint coupling, a measure of similarity is needed. Here, the angular deviation of the first eigenvector is chosen [19]. The standard deviation of this value would be $90°$ in case of no similarity, i.e. statistical independence. For the ten subjects, the standard deviation of the angle is $14.8°$. In order to evaluate the generic applicability of the algorithm, the simulations have all been performed with respect to the worst case, referring to the subject with the least similarity with the average. This ”worst-case” subject is no. 39, whose first PC encloses the maximum angle of $31.9°$ with the mean eigenvector.

This subject’s gait is reconstructed based on motion data of the other nine subjects. To obtain an ”average” matrix $\Gamma$, trajectories of equal length ($n$ samples) of healthy subjects are concatenated in a matrix $\mathbf{D}$, and $\mathbf{D}$ is then analyzed using PCA. Thus, if $d$ DoFs are considered, a $kn \times d$ reference trajectory matrix $\mathbf{D}$ of template subjects is generated, from which the desired quantity $p$ of principal components is derived.




(a) PCA using angle data $\boldsymbol{\varphi}$ (b) PCA using $\boldsymbol{\varphi}, \dot{\boldsymbol{\varphi}}$ (c) PCA using $\boldsymbol{\varphi}, \dot{\boldsymbol{\varphi}}, \ddot{\boldsymbol{\varphi}}$

Fig. 2. Estimates of hip and knee angles $\varphi_h$ and $\varphi_k$ based on the subject's own gait pattern. PCA reconstruction is based on angle data (a), angles+angular velocities (b), angles+angular velocities+angular accelerations (c). The optimal number of principal components used is (from left to right): 2, 4, and 5. Displayed are hip and knee joint angles in the sagittal plane. The dashed blue line is the original trajectory, the black dots indicate estimates.

The results of motion intention estimation with averaged gait data are displayed in Fig. 3. Although hip motion is still estimated quite well, the performance of knee motion estimation deteriorates dramatically compared to the case with known synergies, displayed in Fig. 2. Interestingly, also less PCs are optimal. An explanation is that the similarity in the lower eigenvectors decreases (this hypothesis is confirmed by an increasing angular deviation). This would mean that the common basis for walking, the first eigenvectors, are close for all people, but each person uses additional, less pronounced individual synergies.

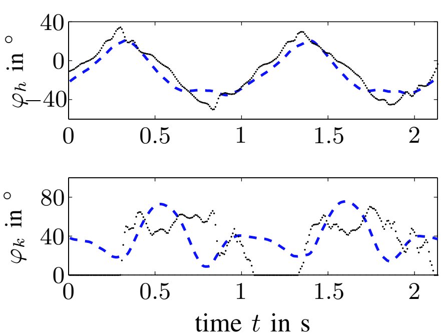



Fig. 3. Estimates of sagittal hip and knee angles $\varphi_h$ and $\varphi_k$ based on averaged reference data. The subject represents the worst case, exhibiting the least similarity with the averaged synergy pattern. PCA reconstruction is based on angles, velocities, and accelerations. The optimal number of eigenvectors used is 4. The dashed blue line is the original trajectory, the black dots show the estimated trajectory.

## V. Kalman Filter

As can be seen from the results of the preceding section, motion intention detection with raw estimated data does not work satisfactorily. However, during the reconstruction procedure, redundant data is generated: accelerations, velocities and angles are estimated. As PCA is completely static and does not account for the relationship between the time derivatives, they are not internally coherent. This redundancy of uncertain estimates can be exploited using a Kalman filter.

### A. Filter Design

The filter is designed for each joint separately based on the simple dynamic model of a double integrator. During filtering, each of the values is corrected so that they become coherent and fit the model. Under the assumption that the errors in the PCA-estimated variables $\varphi_e$, $\dot{\varphi}_e$ and $\ddot{\varphi}_e$ can be modelled by additive uncorrelated noise, better estimates $\hat{\varphi}$, $\hat{\dot{\varphi}}$ and $\hat{\ddot{\varphi}}$ are produced. This design is displayed in Fig. 4. The Kalman filter requires a definition of the uncertainty of each measurement. To assess the value of the uncertainties, i.e. $E(w^2)$, $E(v_1^2)$ and $E(v_2^2)$, a statistical error analysis is performed for the reconstructed gait patterns of all 10 subjects.

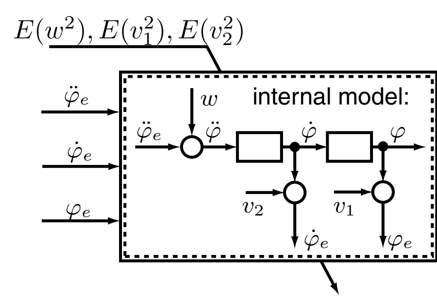



Fig. 4. Design of the Kalman Filter. Based on an internal model, it corrects the noisy values $\varphi_e$, $\dot{\varphi}_e$, $\ddot{\varphi}_e$, which result from the static PCA estimation, and produces dynamically coherent values $\hat{\varphi}$, $\hat{\dot{\varphi}}$, and $\hat{\ddot{\varphi}}$.

### B. Evaluation of the Extended Algorithm

The effect of Kalman filtering is shown exemplarily in Fig. 5, using the estimated gait trajectory of subject 39, whereby the PCA is based on averaged gait, as displayed in Fig. 3. Since the hip trajectory is estimated very well and thus less critical, only the knee angle and angular velocity are displayed. Obviously, the simultaneous correction of the reconstructed time derivatives leads to a considerable enhancement of the estimation.

A rather simple way to further improved motion estimation lies in more suitable reference trajectories. The trajectories used for the simulations presented here have been taken from a very heterogeneous group of subjects, without accounting for the quite different dynamic properties. Therefore, with more similar subjects, estimation results should further improve.

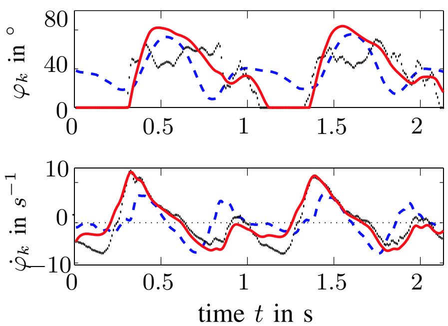



Fig. 5. Kalman filtering of the knee angle $\varphi_k$ and the angular velocity $\dot{\varphi}_k$, which were estimated using PCA based on averaged reference data (as depicted in Fig. 3). The dashed blue line is the original trajectory, the black dots show the "raw" estimation, and the solid red line shows the Kalman-filtered estimation.

## VI. Functional Analysis

Up to this point, the analysis has been performed in joint space, comparing hip and knee trajectories. However, walking is a task-oriented motion. The success of the described method, apart from achieving a physiological motion, depends on its adequacy to fulfill the locomotion task. To evaluate this task-oriented motion, an analysis of the Cartesian foot position is performed. The foot trajectory is displayed in Fig. 6, where the origin of the coordinate system is fixed at the hip, as indicated.

When the subject's own eigenvectors are used, the generated trajectory shows a good similarity to the original gait cycle (considering step-to-step variance present in normal gait). In the case of averaged reference synergy data, estimation errors grow. However, the task-essential stance phase is estimated better than the swing phase. Overall, even this worst-case estimation proves useful to detect the patient's intention, although it is not yet enough to be used as a controller reference trajectory.

One improvement could reside in the augmentation of considered variables in the PCA algorithm, for example by an additional inclusion of Cartesian data or kinetic information (e.g. ground reaction forces).

## VII. Practical Evaluation: Estimation of Knee Motion only

The first practical testbed for the proposed motion intention estimation will be an above-knee prosthetic test setup. In order to be able to perform preliminary experiments with healthy subjects, we plan to use a "fake" prosthesis, where the knee is bent and the lower leg is embedded in an enlarged shaft of an above-knee prosthesis. For these healthy subjects, their individual gait pattern will first be recorded and analyzed. Based on the extracted synergy information, the motion estimation will then work as a controller for the prosthesis.

For the application of above-knee protheses, the hip trajectory of the ipsilateral leg can be used for estimation as well. The results of such an estimation are shown in Fig. 7: A very good motion intention estimation is possible, even if the subject's synergies are not known, as additionally depicted in (a).

## VIII. Conclusion

A general approach to motion intention estimation employing synergy analysis has been presented and applied to two medical example applications. The results indicate that trajectories for inoperable limbs can be generated on-line using motion information of sound limbs. It has also been shown that an estimation using averaged synergy information from healthy subjects is possible.

An important question is the capability of patients to control their leg via the other one, meaning how far a patient can benefit from a coupling scheme of sound and robotically moved DoFs, which is, in contrast to normal synergies, purely unidirectional. This question will be addressed by a practical evaluation.

In order to make the algorithm more generic, a segmentation or clustering of motion might be beneficial, thus allowing for a more detailed and accurate modelling of distinct synergies in different motions. This motion segmentation could be performed by hand (e.g. by division into stance and swing), but also by dynamic clustering, using methodologies such as "Generalized Principal Component Analysis" as suggested by [20].

Future work will also focus on a dynamic extension of the proposed PCA-based motion intention estimation. With exception of the Kalman filter, the estimation is based on instantaneous motion data only. Additional knowledge about the dynamics of the human body shall be integrated by a model-based filter to ensure dynamic consistency of the estimated trajectory [21]. Furthermore, the presented algorithm shall be combined with dynamic identification, which also allows for a predictive dimension.

## IX. Acknowledgments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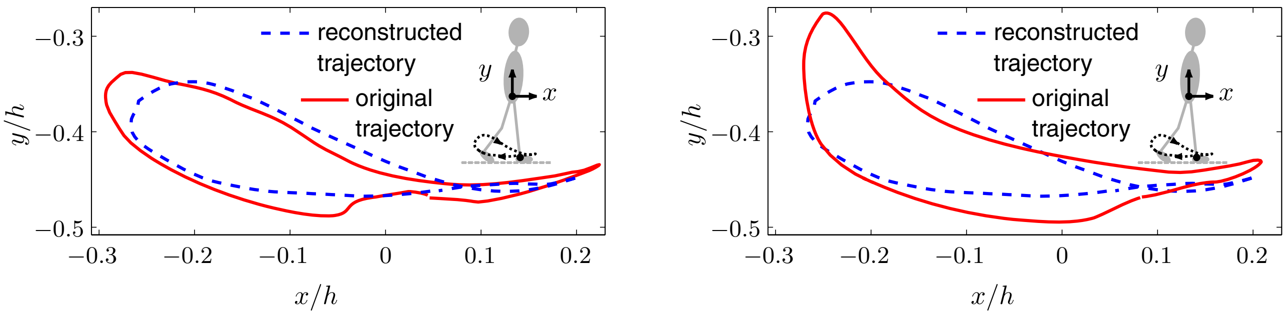
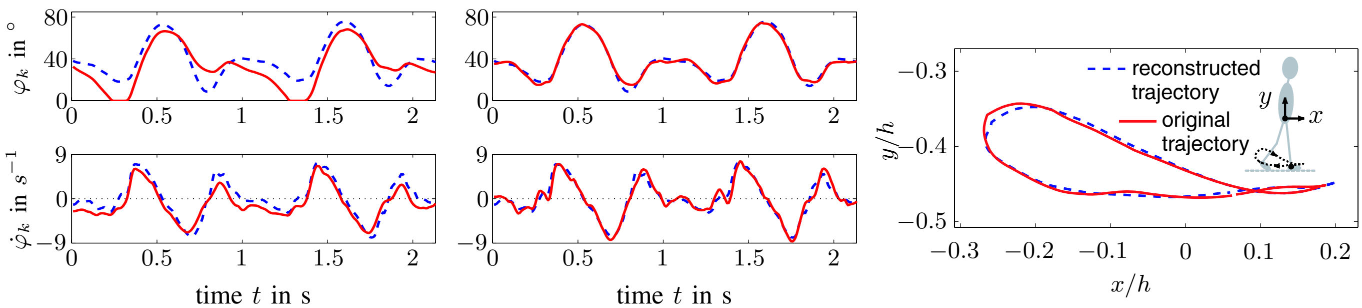



(a) Estimation using the subject's own gait.

(b) Estimation using averaged gait.

Fig. 6. Resulting foot trajectory in the Cartesian hip coordinate system (normed to body height $h$). Displayed are Kalman-filtered estimates using own eigenvectors and averaged eigenvectors, respectively (both estimates are based on angles, velocities and accelerations, as depicted in figures 2 and 3). The dashed blue line is the original trajectory of one gait cycle, the solid red line represents its reconstruction.




(a) Estimation based on average gait

(b) Estimation based on own gait pattern

(c) Cartesian trajectory of case b)

Fig. 7. Estimation of knee motion only. Depicted are estimates of the knee angle and angular velocity based on the subject's own gait pattern (a), and on averaged gait data (b). The optimal number of principal components used is 7 in both cases. The dashed blue line is the original trajectory, and the solid red line shows the Kalman-filtered estimated trajectory.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