Communication Topology Design for Large-scale Interconnected Systems with Time Delay

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***Abstract*— Communication networks provide a larger flexibility with respect to the control design of large-scale interconnected systems by allowing the information exchange between the local controllers of the subsystems. The use of communication networks comes, however, at the price of non-ideal signal tranmission such as time delay which is a source of instability and deteriorates the control performance. This paper introduces an approach for the design of the communication topology for the distributed control of large-scale interconnected systems in order to optimize the whole system's performance in the presence of constant time delay. First, a decentralized control law that stabilizes the overall interconnected system is designed. Then the performance is improved by designing the distributed control law, i.e. allowing the controller of the subsystems to exchange information, by considering the time delay in the networks. As a novelty in this paper, the design of the communication topology between the controllers is also considered. The problem is formulated as a mixed-integer optimization problem. Furthermore, a method based on matrix perturbation theory is discussed to design the topology which also captures the relation between the time delay, controller gain and performance of the overall system. In addition, it is shown that the proposed strategy also guarantees the stability of the overall system under the permanent communication link failure. The results are validated through a numerical example.**

I. Introduction

The design of control algorithms for complex dynamical systems has become a vibrant part of research due to the wide applicability and impact with applications ranging from smart power grids, water distribution and traffic systems to large arrays of micro-electro-mechanical systems (MEMS), formation of vehicles, and sensor-actuator networks.

The key challenge for the control of large-scale dynamical systems is the complexity of the overall system in terms of the number of subsystems and their interconnections. First results addressing the complexity of large-scale systems have been achieved within the decentralized control framework developed since the seventies, see, e.g. [1] for a nice overview. Typically, the performance of decentralized control approaches is degraded compared to centralized control approaches as only the local subsystem information is used for the control.

Digital communication networks allow the communication between the subsystems and thereby provide a larger flexibility with respect to the control design: Instead of only local subsystem information also neighboring subsystems' states can be used for the control. These novel approaches are also known under the notion of *distributed control* [2]. As a result, typically a better performance is achieved compared to the standard decentralized approach [3], [4]. Another merit of using information from the neighboring subsystems is that it can be used to stabilize the interconnected system when no stabilizing decentralized control law exists, for example in the presence of decentralized fixed modes (DFM) in the system [5].

Most of the works in the known literature for distributed systems deal with the problem of stabilization of large-scale systems, e.g. [6], [7]. An approach which combines distributed control and optimization can be found in [8] where the idea of dual decomposition is used for decomposition and distributed optimization of feedback systems. Specifically, distributed optimization is used to iteratively update local controllers of a distributed system based on a gradient approach. The combination of both methods, i.e. optimization and feedback control thereby results in a better system's performance and a guaranteed stability. All of the works mentioned above assume that the communication topology is given a priori. The introduction of a communication network, however, provides an additional degree of freedom for the structural design of the distributed controller in terms of the communication topology. The problem of designing communication topology has been studied in the area of sensor networks where the goal is to minimize the sensors' power consumption while maintaining the connectivity of the network, e.g. in [9].

The communication topology also plays an important role in the performance and stability of the controlled overall system as indicated by several results especially in the area of multi-agent systems. For example, in [10] it is shown that the connectivity of the agents could improve the convergence rate of the whole system to attain a desired behavior while decreasing the robustness of the systems to the time delay. The authors in [11] present the design of the topology of a random sensor network to maximize the convergence rate of the consensus algorithm. The authors in [12] consider the synchronization of identical networked systems by introducing a distributed controller where the controller gain and structure are optimized subject to a given constraint. Furthermore, the authors in [13] present an approach to design both the controller gain and the communication topology between the controllers thereby exploring the additional degrees of freedom offered by the communication network. The problem is formulated as a mixed integer optimization problem. Moreover, the approach also guarantees the stability, i.e. increases the robustness of

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the overall system in the presence of permanent communication link failures. However, all of the works mentioned above assumed an ideal communication network. The use of communication networks comes, however, at the price of non-ideal signal transmission: the data sent through the networks experience time delay or suffer transmission data losses which is a source of instability and deteriorates the control performance [14].

In this paper, we consider the design of communication topology between the controllers of interconnected system in the presence of time delay in the communication networks. The time delay is assumed to be identical and constant for all communication links. The goal is to improve the performance of the interconnected system and in addition increase the robustness w.r.t. the permanent link failures. In order to achieve the goals, we extend our design approach in [13] to incorporate constant and identical time delay. First, we design the decentralized control law to stabilize the overall interconnected system. Then, we improve the performance of the systems by designing a controller which uses the state information from other subsystems under given communication network constraints and network induced time delay. As a performance metric we consider the convergence rate.

The remainder of the paper is organized as follows. The problem formulation is described in Section II. The design approach for ideal communication networks is reviewed in Section III. The design of the distributed control law in order to improve the performance of the systems under a given time delay is considered in Section IV where the problem is formulated as a mixed integer optimization problem. A method based on the matrix perturbation theory is discussed in Section V. The proposed approach is demonstrated through a numerical example in Section VI.

II. Problem Formulation

Consider an interconnected system of N heterogenous linear time invariant (LTI) subsystems described by the following differential equations

$$\dot{x}_i = A_i x_i + \sum_{j\in N_i} A_{ij} x_j + B_i u_i, \tag{1}$$

where $i = 1,2,...,N$ denotes the $i-$th subsystem, $x_i \in \mathbb{R}^{n\times 1}$, $u_i \in \mathbb{R}^p$ are the state of subsystem i and the control input to subsystem i, and $A_i, A_{ij} \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times p}$. The term $\sum_{j\in N_i} A_{ij} x_j$ represents the physical interconnection between the subsystems where N_i is the set of subsystems to which subsystem i is physically connected. The interconnected system described above can be found for example in power grids. Here we consider a state feedback controller for which the control law can be written as

$$u_i = K_i x_i + \sum_{j\in G_i} K_{ij} x_j(t-\tau), \tag{2}$$

which is known as *distributed* control law since the controller for each subsystem does not only depend on its own states but also the states of the other subsystems. Here G_i represents a set of subsystems to which controller i could communicate,

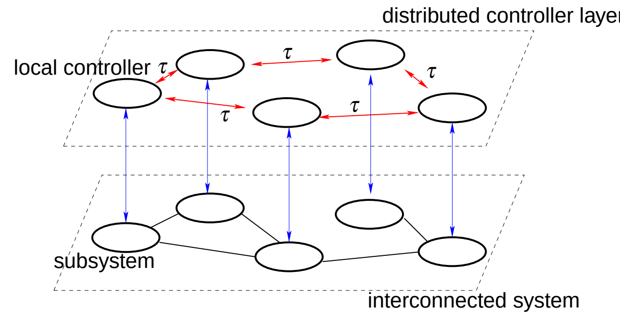

Fig. 1. Interconnected system with distributed control architecture. The communication network has a constant and identical time-delay τ.

i.e. exchange information. In this paper, we consider a constant and identical time delay τ for all communication links. The interconnected system with its distributed controller is illustrated in Fig. 1. If $K_{ij} = 0, \forall i$ and $\forall j \in G_i$, then the control law is called a *decentralized* control law.

In this paper, first we address the following practically relevant question: given a pre-designed distributed controller under a given communication network constraint and without consideration of time delay, what is the effect of time delay on the system performance? Specifically, we would like to investigate up to which time delay value a communication network is still beneficial in terms of the convergence rate of the overall system. Next, given a constant, identical time delay for all communication links, we aim at designing the distributed control (2), if any, such that the performance of the whole system is improved and the stability of the system is guaranteed. Moreover, as an additional goal, the robustness of the interconnected system w.r.t. the permanent link failures has to be guaranteed. In order to achieve the goals, we extend our approach in [13] to incorporate constant and identical time delay. First we design a decentralized control law that guarantees the stability of the interconnected system. The performance of the systems is then improved by designing a distributed control law, i.e. the second term of (2).

III. Distributed Control Design

In this section we review the procedure for designing the distributed controller together with the communication topology proposed in [13] in order to improve the performance of the systems without consideration of time delay. As a performance metric, the convergence rate of the overall system is considered.

A. Decentralized control design

As a design strategy, first we design the decentralized control law using the standard methods, e.g. [6] that stabilizes the interconnected system (1), that is, we consider the decentralized controller synthesis for the interconnected system (1) using the standard method with the control input given by

$$\tilde{u}_i = K_i x_i. \tag{3}$$

Remark 1: It is assumed that all the decentralized fixed modes, if any, are in the open left half plane. The problem with unstable DFMs is the subject of future works.

Let $A_i + B_i K_i = \bar{A}_i$. The closed loop expression of the interconnected system (1) with the decentralized control law can be written as

$$\dot{x} = A_{\text{dec}} x, \; x(t_0) = x_0, \tag{4}$$

where $x = [x_1, x_2, \cdots, x_N]^T$ and

$$A_{\text{dec}} = \begin{bmatrix} \bar{A}_1 & A_{12} & \cdots & A_{1N} \\ A_{21} & \bar{A}_2 & \cdots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \cdots & \bar{A}_N \end{bmatrix} \in \mathbb{R}^{nN \times nN}.$$

It is well known that the solution of (4) is given by $x(t) = e^{A_{\text{dec}}(t-t_0)} x_0$ and the state norm satisfies

$$\|x(t)\| \leq e^{\|A_{\text{dec}}\|(t-t_0)} \|x_0\|, \forall t \geq t_0, \tag{5}$$

and

$$\|x(t)\| \leq e^{\text{Re}\{\lambda_{\max}\}(t-t_0)} \|x_0\|, \forall t \geq t_0, \tag{6}$$

where $\text{Re}\{\lambda_{\max}\}$ represents the real part of the largest eigenvalues of A_{dec}.

B. Distributed controller design

Assume that the decentralized control law stabilizing the interconnected system (1), that is the first term of (2), has been designed. Next we will improve the performance of the systems for a certain performance metric by designing the distributed controller, that is the feedback gain and the communication topology under a given communication constraint without consideration of time delay, i.e. $\tau = 0$.

The objective is to improve the performance of the overall system, i.e. increase the convergence rate by designing the second term of (2) given by the following controller

$$\hat{u}_i = \sum_{j \in G_i} d_{ij} K_{ij} x_j, \tag{7}$$

where $d_{ij} \in \{0, 1\}$ is a binary number that shows the possibility to perform the state information exchange between controller i and j, that is $d_{ij} = 1$ means that a communication link is added between controllers i and j and vice versa. The new closed loop expression of (1) with the addition of controller (7) is given by

$$\dot{x} = \bar{A} x, \; x(t_0) = x_0, \tag{8}$$

$$\begin{aligned} \bar{A} &= \begin{bmatrix} \bar{A}_1 & A_{12} & \cdots & A_{1N} \\ A_{21} & \bar{A}_2 & \cdots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \cdots & \bar{A}_N \end{bmatrix} + \begin{bmatrix} 0 & \bar{A}_{12} & \cdots & \bar{A}_{1N} \\ \bar{A}_{21} & 0 & \cdots & \bar{A}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{A}_{N1} & \bar{A}_{N2} & \cdots & 0 \end{bmatrix}, \\ \bar{A} &= A_{\text{dec}} + A_{\text{dist}}. \end{aligned}$$

The matrix A_{dec} is the closed-loop system with the decentralized control, thus, the matrix A_{dec} is stable. The term $\bar{A}_{ij}$ is defined as $\bar{A}_{ij} = d_{ij} B_i K_{ij}$. Furthermore, we assume that not arbitrary many links can be added, i.e. the number is limited by an upper bound induced by the communication constraint

$$\sum_{1 \leq i \leq j \leq N} \gamma_{ij} d_{ij} \leq c, \tag{9}$$

where $c > 0$ is the total cost constraint on the communication network, and γ_{ij} represents a cost to establish a link between subsystem i and j. This cost is typically related to factors such as the distance between the subsystems.

The problem can then be formulated as finding the gain and topology of the distributed controller such that the convergence rate of the overall interconnected system is optimized under a given communication constraint. The distributed controller is given by the following proposition.

Proposition 3.1: [13] Consider an interconnected system (8). If there exists a solution to the optimization problem

$$\begin{aligned} \underset{K_{ij}, d_{ij}}{\text{minimize}} \quad & \text{Re}\{\lambda_{\max}(\bar{A})\} \\ \text{subject to} \quad & \text{Re}\{\lambda_{\max}(\bar{A})\} < \text{Re}\{\lambda_{\max}(A_{\text{dec}})\}, \\ & \sum_{1 \leq i \leq j \leq N} \gamma_{ij} d_{ij} \leq c, \\ & d_{ij} \in \{0, 1\}, \end{aligned} \tag{10}$$

then the convergence rate of the interconnected system with the distributed control law (2) is higher than with the decentralized control law (3) and the whole system remains stable.

The optimization problem (10) is a mixed integer optimization problem since it is solved with respect to both the feedback gain and the communication topology of the distributed controller. The optimization (10) can be solved using well-known techniques such as relaxation and decomposition techniques or cutting planes approaches [15].

Remark 2: Since the first term of (2) is designed in advance and not part of the decision variables of the optimization problem (10), the performance may not be optimal. However, as will be shown later, the stability of the overall system is guaranteed in the presence of permanent communication link failures.

IV. Topology design under time delay

In this section, we consider a more realistic scenario by considering a non-ideal communication networks where the information exchange is afflicted by time delay τ which is assumed to be constant and identical for all communication links. From (2), the closed loop expression (1) with constant and identical time delay τ can be written as

$$\begin{aligned} \dot{x}(t) &= A_{\text{dec}} x(t) + A_{\text{dist}} x(t-\tau), \\ x(\theta) &= \mathbf{x}_0, \; \forall \theta \in [-\tau, 0]. \end{aligned} \tag{11}$$

Before proceeding, we recall the following theorem on the exponential stability of time delay system [16]. For the sake of convenience, we adapt the notation in [16] into ours.

Consider the time-delay systems (11) utilizing the following transformation

$$z(t) = \mathrm{e}^{\alpha t} x(t), \tag{12}$$

where $\alpha > 0$ is the delay decay rate, that is the convergence rate of time-delay system (11), to transform (11) into

$$\dot{z}(t) = (A_{\text{dec}} + \alpha I) z(t) + A_{\text{dist}} \mathrm{e}^{\alpha \tau} z(t-\tau). \tag{13}$$

Theorem 4.1: [16] Consider the time-delay system (13) with delay time $\tau > 0$ and delay decay rate α. This system is exponential stable with decay rate α if there exists symmetric and positive-definite matrices $P > 0, Q > 0$ such that the following inequality holds

$$S_1 = \begin{bmatrix} \hat{A}^T P + P\hat{A} + \tau Q & \tau e^{\alpha\tau} \hat{A}^T P A_{\text{dist}} \\ \tau e^{\alpha\tau} A_{\text{dist}}^T P \hat{A} & -\tau Q \end{bmatrix} < 0, \quad (14)$$

where $\hat{A} = A_{\text{dec}} + \alpha I + A_{\text{dist}} e^{\alpha\tau}$.

A. Performance-guaranteed time-delay

Since time delay deteriorates the performance of the overall system in general, first we address the following practically relevant question: given a pre-designed distributed controller under a given communication network constraint and without consideration of time delay using the approach in Section III, up to which time delay value a communication network is still beneficial in terms of the convergence rate of the overall system. In this paper, we refer to the corresponding time delay as a performance-guaranteed time delay bound. Specifically, the performance-guaranteed time delay bound $\tau_{max} \in [0, \infty)$ is defined as the maximum time delay

$$\begin{aligned} &\max \tau \\ &\text{s.t. } \alpha \geq |\text{Re}\{\lambda_{\max}(A_{\text{dec}})\}| \end{aligned} \quad (15)$$

where $\text{Re}\{\lambda_{\max}(A_{\text{dec}})\}$ denotes the decay rate of the interconnected system with the decentralized control law.

From Theorem 4.1 and (15), the performance-guaranteed time delay bound $\tau_{\max}$ is given by the solution to the following optimization problem

$$\begin{aligned} &\underset{\tau}{\text{maximize}} \quad \tau \\ &\text{subject to} \quad S_1 < 0 \text{ with } \alpha = |\text{Re}\{\lambda_{\max}(A_{\text{dec}})\}|. \end{aligned} \quad (16)$$

We set $\alpha = |\text{Re}\{\lambda_{\max}(A_{\text{dec}})\}|$ which is the convergence rate of the overall system with the decentralized control law (3) since we want the convergence rate by using the distributed control law (2) to be higher than with the decentralized one. In particular, the performance-guaranteed time delay bound can be calculated by increasing τ until the positive definiteness conditions of P, Q are violated.

Remark 3: A less conservative result on the performance-guaranteed time delay bound $\tau_{\max}$ can be achieved using the result on the exponential stability of time-delay system, e.g. [17].

B. Distributed controller design

Next, we consider the following problem: given a constant, identical time delay τ for all communication links, design the distributed control law (2), if any, such that the performance of the whole system is improved and the stability of the system is guaranteed. Combining Theorem 4.1 and Proposition 3.1, the distributed controller design is given by

Proposition 4.1: Consider an interconnected time delay system (11) with a given constant and identical time delay τ for all communication links. If there exists a solution to the optimization problem

$$\begin{aligned} &\underset{K_{ij}, d_{ij}}{\text{minimize}} \quad \alpha \\ &\text{subject to} \quad S_1 < 0, \\ &\qquad \alpha > |\text{Re}\{\lambda_{\max}(A_{\text{dec}})\}|, \\ &\qquad \sum_{1 \leq i \leq j \leq N} \gamma_{ij} d_{ij} \leq c, \\ &\qquad d_{ij} \in \{0, 1\}. \end{aligned} \quad (17)$$

then the convergence rate of the interconnected time delay system (11) with the distributed control law (2) is higher than with the decentralized control law (3) and the whole system remains stable.

The proof is straightforward from Theorem 4.1 and Proposition 3.1.

Remark 4: The first constraint in (17) is not an LMI problem due to the nonlinear terms $P\hat{A}$ and $\hat{A}PA_{\text{dist}}$ in (14). However, the optimization problem can be solved using numerical BMI techniques.

Assumed that $\gamma_{ij} = 1$. The complexity of the design approach for a given communcation links c in terms of the number of combinations that has to be carried out is given by $\binom{c_{\max}}{c} = \frac{c_{\max}!}{c!(c_{\max}-c)!}$ where $c_{\max} = \frac{N}{2}(N-1)$. Thus, it may become very hard to solve (17) for large number of subsystems. Next we propose a method in order to add the communication link based on matrix perturbation theory.

V. Where to add the communication links?

In this section we present a method to select which controllers should communicate in order to improve the performance of the overall system for a given number of available communication links. The idea is to use matrix perturbation theory in order to investigate how the structure of the distributed controller influences the largest eigenvalue of the overall system. Furthermore, for the sake of simplicity, we constrain ourselves for the remainder of this paper by the following assumptions:

- A1 The subsystems are scalar, i.e. $x_i \in \mathbb{R}$
- A2 The number of communication link $c = 1$ with $\gamma_{ij} = 1$
- A3 The communication is bidirectional
- A4 The distributed controller gain K is fixed and equal for all subsystems which results in $A_{\text{dist}}^T = A_{\text{dist}}$.

We introduce assumption A4 since we are interested in the design of the topology and in order to investigate the influence of the controller gain on the overall system performance. Moreover, the approach described in the following can also be extended to the case when $x_i \in \mathbb{R}^n$ and for multiple communication links, i.e. $c > 1$ which however results in a more complicated formulation.

The real part of the rightmost eigenvalues of (11), i.e. $\text{Re}\{\lambda_{\max}\}$ determines the decay rate of the whole interconnected system. Thus, we are interested in the minimization of the real part of the rightmost eigenvalue $\lambda_{\max}$. In this section, the eigenvalue sensitivity is analyzed in order to investigate how the structure of the distributed control law affects $\lambda_{\max}$ of (11). Eigenvalue sensitivity gives an insight on the behavior of the eigenvalues of a matrix when the matrix is perturbed. Moreover, the magnitude of the eigenvalue sensitivity informs about the size of the eigenvalue displacement in the complex plane. The eigenvalues of (11) are equivalent to the roots of the characteristic equation

$$\det(\lambda I - A_{\text{dec}} - A_{\text{dist}}\,\mathrm{e}^{\lambda\tau}) = 0. \tag{18}$$

Note that eq. (18) is a nonlinear eigenvalue problem and has infinitely many solutions. However, the number of eigenvalues to the right of any vertical line, $\mathrm{Re}(\lambda) \geq r$, with $r \in \mathbb{R}$, is finite, and hence $-\infty$ is the only accumulation point for the real parts of the eigenvalues [20]. First we recall the sensitivity analysis for the nonlinear eigenvalue problem [18]. Consider a nonlinear eigenvalue problem depending on a parameter h, G_h. Hence the sensitivity of a solution to the nonlinear eigenvalue problem λ, which is the generalization for the linear case is given by [18]

$$\lambda'(h) = \frac{v^* \frac{dG_h}{dh}(\lambda) y}{v^*(I - \frac{dG_h}{d\lambda}(\lambda))y}, \tag{19}$$

where v and y are the left and right eigenvector respectively with normalization $v^* y = 1$ where v^* is the complex conjugate transpose of v. Since the goal is to find the communication topology such that the convergence rate of the overall system (11) with the distributed control law is higher than the one with the decentralized controller, we would like to find the structure of the perturbation to the rightmost eigenvalues $\lambda_{\max}$ of (11) such that its sensitivity is negative and the magnitude is maximum. Note that the rightmost eigenvalues of (18) can be computed numerically using, e.g. [19]. The function $G_K(\lambda)$ is given as

$$G_K(\lambda) = A_{\text{dec}} + A_{\text{dist}}(K)e^{-\tau\lambda}. \tag{20}$$

The sensitivity of the rightmost eigenvalues of (11) is

Lemma 5.1: Consider an interconnected system (11) under assumption A1-A4. The sensitivity of the rightmost eigenvalue $\lambda_{\max}$ w.r.t. the structure of the distributed control, i.e. the communication topology is given by

$$\lambda'_{\max} = \frac{(v^*_{r_i} y_{r_j} + v^*_{r_j} y_{r_i})\ \mathrm{sign}(K)}{e^{\tau\lambda_{\max}} + K\tau(v^*_{r_i} y_{r_j} + v^*_{r_j} y_{r_i})} \tag{21}$$

where v_r, y_r are the left and right eigenvector w.r.t. $\lambda_{\max}$.

Proof: Assume that the perturbation K works on $\bar{A}_{ij}$ and $\bar{A}_{ji}$ of A_{dist}. From (19) with $G_K(\lambda)$ given by (20), the sensitivity of $\lambda_{\max}$ can be written as

$$\lambda'_{\max}(K) = \frac{v^*_r \frac{dG_K}{dK}(\lambda_{\max}) y_r}{v^*_r(I - \frac{dG_K}{d\lambda_{\max}}(\lambda_{\max}))y_r}, \tag{22}$$

with $\left[\frac{dG_K}{dK}\right]_{ij} = \left[\frac{dG_K}{dK}\right]_{ji} = \mathrm{sign}(K)e^{-\tau\lambda_{\max}}$ and zero otherwise. Moreover, $\left[\frac{dG_K}{d\lambda_{\max}}\right]_{ij} = \left[\frac{dG_K}{d\lambda_{\max}}\right]_{ji} = K\tau e^{-\tau\lambda_{\max}}$ and zero otherwise. After some straightforward computation, we have

$$\lambda'_{\max} = \frac{e^{-\tau\lambda_{\max}}(v^*_{r_i} y_{r_j} + v^*_{r_j} y_{r_i})\ \mathrm{sign}(K)}{\sum y_{r_i} v^*_{r_i} + K\tau e^{\tau\lambda_{\max}}(v^*_{r_i} y_{r_j} + v^*_{r_j} y_{r_i})}. \tag{23}$$

Since $v^*_r y_r = 1$ and after some calculation, we have

$$\lambda'_{\max} = \frac{(v^*_{r_i} y_{r_j} + v^*_{r_j} y_{r_i})\ \mathrm{sign}(K)}{e^{\tau\lambda_{\max}} + K\tau(v^*_{r_i} y_{r_j} + v^*_{r_j} y_{r_i})}. \tag{24}$$

■

As can be observed from (21), the sensitivity of the rightmost eigenvalue depends on the distributed controller gain K, time delay τ and also the elements of the eigenvectors corresponding to the rightmost eigenvalue $\lambda_{\max}$. When the rightmost eigenvalue is complex, only the movement along the real axis, i.e. $\mathrm{Re}\{\lambda'_{\max}\}$ is needed to be considered. The communication link is then added according to

Proposition 5.1: Consider an interconnected system (11) under assumption A1-A4 with a constant time delay τ. The convergence rate of the overall system with distributed control law (2) is maximized by adding a communication link between subsystem i and j which is the solution to

$$\begin{aligned} &\underset{i,j}{\text{maximize}} \quad |\mathrm{Re}\{\lambda'_{\max}\}| \\ &\text{subject to} \quad \mathrm{Re}\{\lambda'_{\max}\} < 0. \end{aligned}$$

Proof: Since we want to have the real part of the rightmost eigenvalue of (11) to be smaller than the one with the decentralized control law, it is required that $\mathrm{Re}\{\lambda'_{\max}\} < 0$. Moreover, the movement of the real part of the perturbed rightmost eigenvalue has to be maximum in order to have the highest convergence rate of (11). ■

Next we have the following result on the stability in the presence of permanent communication link failures.

Corollary 5.1: The stability of the interconnected system (11) is guaranteed under any combination of permanent communication link failures.

Proof: Since the convergence rate of the interconnected system (11) with distributed control law (2) is higher than with the decentralized control law (3), we have $\mathrm{Re}\{\lambda_{\max}(\bar{A})\} < \mathrm{Re}\{\lambda_{\max}(A_{\text{dec}})\} < 0$. Furthermore, due to the local continuity of each eigenvalue w.r.t. the paramater [20], the rightmost eigenvalue of the overall system under any combination of communication link failures, i.e. $\mathrm{Re}\{\lambda_{\max}(\tilde{A})\}$ will be $\mathrm{Re}\{\lambda_{\max}(\bar{A})\} < \mathrm{Re}\{\lambda_{\max}(\tilde{A})\} < \mathrm{Re}\{\lambda_{\max}(A_{\text{dec}})\} < 0$. ■

VI. Numerical Example

We consider an interconnected system consisting of 10 scalar subsystems as shown in Fig. 2. Assumed that $\gamma_{ij} = 1$, time delay $\tau = 0.01$ and the distributed control gain is set to 1. The closed loop system with the decentralized control law, i.e. A_{dec} in eq. (4) is given by

$$\begin{bmatrix} -5 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & -25 & 6 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & -3 & 1 & 0 & 0 & 2 & 0 & 0 & 0 \\ 3 & 0 & 1 & -10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 3 & 0 & 0 & -6 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -7 & 0 & 0 & 0 & 8 \\ 0 & 0 & 2 & 0 & 0 & 0 & -9 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8 & 2 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & -15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 5 & 0 & -20 \end{bmatrix}.$$

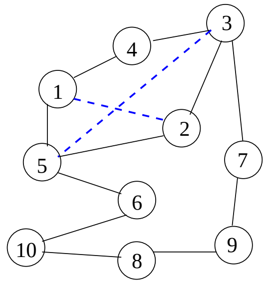

Fig. 2. Interconnection system consists of ten subsystems. The solid and dash lines represent the physical interconnection and the optimal communication topology for number of links equal to 2 respectively.

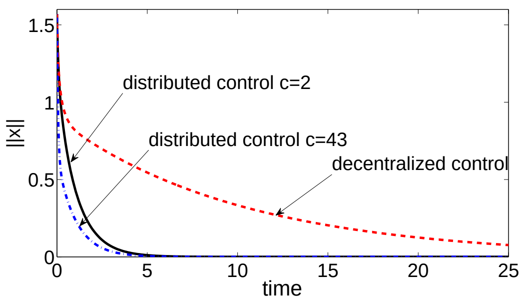

Fig. 3. The convergence of the interconnected system with the distributed control law and the decentralized control law.

First we consider the case when $c = 2$. The optimal communication topology is shown in Fig. 2. As can be observed from Fig. 3, the interconnected system with the distributed control law (2) converges faster than with the decentralized control law (3). Furthermore, by solving the optimization problem (16) using the YALMIP toolbox [21], the performance-guaranteed time delay bound for the resulting topology is $\tau_{\max} = 0.91$. Next we compute the optimal topology for $c = 43$. The optimal topology is achieved without connecting local controller of subsystems 3,5 and 3,6 from 45 links to make a full graph while the performance is shown in Fig. 3. Moreover, the time delay bound $\tau_{\max} = 0.11$. As indicated from the results, having more communication links may result in a lower $\tau_{\max}$ which also indicates that the number of links should be reduced when τ becomes large.

VII. Conclusion and Future Works

This paper introduces an approach for the joint design of the controller gain-communication topology for the distributed control of large-scale interconnected systems in order to optimize the overall system's performance in the presence of constant time delay and guarantee its stability when the communication links are failed. After stabilizing the overall system using a decentralized control law, the performance is then improved by designing the distributed control law, including the communication topology, by also considering the time delay in the networks. The problem is formulated as a mixed-integer optimization problem. Furthermore, a method based on matrix perturbation theory is discussed in order to design the communication topology. The proposed method also provides information on how the time delay and controller gain influence the performance of the overall system. The ongoing work is focusing on solving the optimization problem in a distributed manner in order to reduce the design complexity of the proposed approach and to make it scalable with the number of the subsystems.

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