パケットロスを考慮したネットワーク化システムのQoS制御

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Quality-of-Service Control of Networked Systems with Packet Loss

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Abstract– The stability and performance of a networked control system (NCS) strongly depends on the communication quality, e.g. of the packet loss. Based on the Quality-of-Service (QoS) communication concept we assume the packet loss probability to be adjustable in order to achieve a trade-off between control performance and network cost. In this paper we study the properties and benefits with a periodic adjustment of the packet loss probability. Necessary and sufficient stability conditions are derived, a necessary condition in terms of a lower bound for the loss probability for stabilizing an unstable plant is given. The minimum data rate over a non-dropping link to satisfy a desired deterministic convergence rate is derived. Finally, we show that a periodic switching scheme may improve the state convergence rate. The results are validated in simulations.

Key Words: Networked control system, multiple virtual channels, periodic switching

1 Introduction

Networked control systems (NCSs) are systems whose sensors, actuators, estimation units and control units are connected through a communication network. Compared to point to point interconnection between the plant and the controller, NCSs have advantages such as greater flexibility compared to traditional control systems, reduced wiring, and also permits greater agility in diagnosis and maintenance procedures. However, the signal transmission in a NCS can no longer be regarded as ideal. Time delay and packet loss deteriorate the performance and may destabilize the system.

In this paper, we consider the Quality of Service (QoS) which refers to the capability of a network to provide different communication quality to different network traffic. The QoS control approach of NCS in this paper is based on switching system as proposed in 1) for the presence of piecewise constant time delay where the time delay is considered to be adjustable. In this paper we consider the QoS control scheme for the first time with packet loss, the time delay issue is neglected.

Choosing a low packet loss probability leads to good control performance, however, is considered to be expensive in terms of network cost. For instance, in wireless networks, one way to reduce the probability of erronous transmission is to increase the transmission power at the cost of higher energy consumption. High loss probabilities, on the contrary, results in poor control performance. The goal is to find a trade-off between network cost and control performance.

The influence of packet loss to the stability has been studied e.g in 2), 3), 4) where the communication channel is described as a single channel. In this paper, the network connecting plant and controller is modelled by multiple virtual channels that have different packet loss probabilities (qualities). One can manipulate the packet loss probability by switching from one channel to another. The goal is to find the switching sequences that results in some optimal trade-off between network cost and control performance. For the simplicity of design and application we consider an open loop switching scheme, i.e. the switching sequence does not depend on the system state. Moreover, we consider the switching scheme to be periodic, resulting in a periodic variation of the packet loss probability. The stabilization of NCS with periodic switching is also studied in 5) for system with multiple sensors, actuators and a shared communication channel.

First we derive necessary and sufficient stability conditions, and a necessary condition in terms of a lower bound for the loss probability for stabilizing an unstable plant. The minimum data rate over a non-dropping link to satisfy a desired deterministic convergence rate is derived. Finally, we show that a periodic switching scheme may improve the state convergence rate. The results are validated in simulations.

The remainder of this paper is organized as follows. The problem setting is presented in Section 2, Stability is investigated in Section 3, performance is studied for two different scenarios in Section 4 followed by simulations in Section 5.

2 Problem Formulation

Let's consider the feedback control system in Fig.1 where the plant H has a state-space equation in discrete-time of the following form :

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k, \\ y_k &= Cx_k, \end{aligned} \tag{1}$$

where $x \in \mathcal{R}^n$ is the state, $u_k \in \mathcal{R}^m$ is the control input, $y_k \in \mathcal{R}^p$ is the observed system output. Assume that (A, B) is controllable and (A, C) is observable . For the simplicity of notation, in this paper we consider a single-input system. The extension to the multi-input case is straightforward. The plant is connected to the controller K which is constant

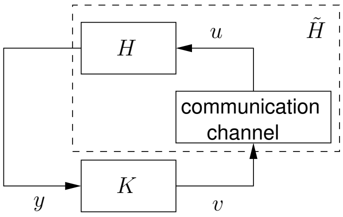


Fig. 1: Networked control system (NCS).

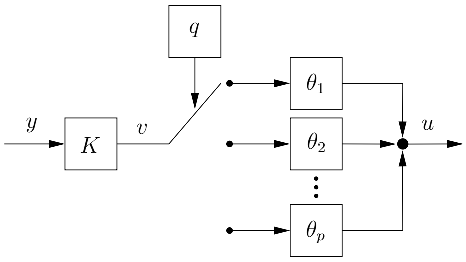


Fig. 2: Model of communication channel with QoS control: multiple virtual channels with different loss probabilities $\alpha_i \neq \alpha_j$ if $i \neq j$, $i, j \in I_p$, and switching indicator q

($K \in \mathcal{R}^{0\times 0}$) over a communication channel which is modelled by p virtual channels as shown in Fig.2. We consider each of these virtual channels as a lossy channel modeled by a stochastic process $\theta_{i,k} \in \{0, 1\}$. Here $\theta_{i,k} = 0$ means that the packet at time k being transmitted via the i-th channel is lost; otherwise it arrives. Assume that the packet loss process is Bernoulli process with probability

$$\alpha_i := Prob\{\theta_{i,k} = 0\}.$$

We then define Θ_k as a vector of the packet loss indicator of all channels which is written as :

$$\mathbf{\Theta_k} = \begin{bmatrix} \theta_{1,k} \\ \vdots \\ \theta_{p,k} \end{bmatrix}.$$

The p virtual channels are switched periodically with the period of N. The notation of the switching pattern is borrowed from 5). Let the vector $q \in I_p^N$ be the switching pattern in one period with $I_p := \{1, 2, ..., p\}$ the index set. The channel used to transmit the packet at time k is the channel indexed by $q(\text{mod}(k, N) + 1)$. For example, with $p = 3$ virtual channels, a period of length $N = 4$, the switching pattern $q = [1, 3, 3, 2]$ means that first channel 1 is used, the two times channel 3, and finally channel 2. This pattern is continuously repeated. Define the channel indicator matrix Q_k as

$$Q_k := [0...010...0]^T \in \mathcal{R}^{p\times 1}, \text{ if } q(\text{mod}(k, N) + 1) = i.$$

Here the value of the i-th element of matrix Q_k is 1 if the i-th channel is used, all others are zero. Note that Q_k is a N-periodic matrix.

In the further analysis the extended plant $\tilde{H}$ is considered, containing the original plant H (1), and the model of the communication channel with the channel indicator matrix Q_k and the packet loss indicator Θ_k; see also Fig.1. Furthermore we consider the state feedback controller case, i.e. C the identity matrix

$$\begin{aligned} x_{k+1} &= Ax_k + B\Theta_k^T Q_k v_k, \\ y_k &= x_k \end{aligned} \tag{2}$$

being a periodic stochastic system. The closed-loop equation of (2) can be written as:

$$x_{k+1} = (A + B\Theta_k^T Q_k K)x_k. \tag{3}$$

To characterize the stability, the notion of mean square stability is used. The origin of (2) is said to be mean-square stable if for any initial condition (x_0, θ_0),

$$\lim_{k\to\infty} E[\|x_k\|^2 \mid x_0, \theta_0] = 0$$

holds.

The main goal of this paper is to derive stability conditions for the periodic stochastic system for a given controller K and to find the minimum packet loss probability α_i such that there exists a controller that stabilizes the overall system in Fig.1.

3 Stability

The closed-loop equation in (3) can be generalized as:

$$x_{k+1} = A_{k,\theta(k)}x_k, \tag{4}$$

where $A_{k,0}$ means that the packet is lost at time k and $A_{k,1}$ indicates that the packet is arrived. Matrix A is a periodic matrix with length N i.e $A_{k+N,\theta_k} = A_{k,\theta_k}$.

First we mention the well known result for the stability of the periodic system.

Lemma 3.1. 6) *For system $x_{k+1} = A_k x_k$ where $A(\cdot)$ is a periodic matrix, the following statements are equivalent each other.*

- *$A(\cdot)$ is stable.*
- *There exists a N-periodic positive definite solution $P(\cdot)$ of the Lyapunov inequality*

$$A_k^T P_{k+1} A_k - P_k < 0.$$

Here $A(\cdot)$ is stable means that the eigenvalues of all multipliers of matrices A are located inside the unit disk in the complex plane.

We now derive stability condition for periodic stochastic system in (4). Assume that each channel is used n_i times in one period ($\sum_{i=1}^{p} n_i = N$).

Proposition 3.1. *For the system in (4), the origin is mean-square stable iff there exists an N-periodic matrix $P_l \in \mathcal{R}^{n\times n}$ such that $P_l = P_l^T > 0$ and*

$$\sum_{i=0}^{1} \alpha_{l,i} A_{l,i}^T P_{l+1} A_{l,i} - P_l < 0 \textit{ for } l \in I_N, \tag{5}$$

where $I_N := \{0, ..., N-1\}$.

Proof. Let‘s define the lyapunov function as:

$$V_k = x_k^T P_k x_k,$$

where P is time varying and $P_k^T = P_k$. The system is mean-square stable iff $E[\Delta V_k \mid x_k, \theta_k] < 0$. If $\sum_{i=0}^{1} \alpha_{l,i} A_{l,i}^T P_{l+1} A_{l,i} - P_l < 0$ for $l \in I_N$ holds for I, then it also holds for $I + N$. Therefore it is sufficient to prove only for one period N. By applying lemma 3.1 and computing the expected value of the difference gives us :

$$\begin{aligned} E[\Delta V] &= E[V_{k+1} \mid x_k, \theta] - V_k \\ &= E[x_k^T A_{k,i}^T P_{k+1} A_{k,i} x_k \mid x_k, i] - x_k^T P_k x_k \\ &= x_k^T E[A_{k,i}^T P_{k+1} A_{k,i} \mid i] x_k - x_k^T P_k x_k \\ &= x_k^T (\alpha_k A_{k,0}^T P_{k+1} A_{k,0} + (1-\alpha_k) A_{k,1}^T \\ &\quad P_{k+1} A_{k,1}) x_k - x_k^T P_k x_k \\ &= x_k^T (\sum_{i=0}^{1} \alpha_{k,i} A_{k,i}^T P_{k+1} A_{k,i} - P_k) x_k < 0. \end{aligned}$$

Therefore, the periodic system is mean-square stable iff there exists N-periodic P_k such that

$$\sum_{i=0}^{1} \alpha_{l,i} A_{l,i}^T P_{l+1} A_{l,i} - P_l < 0$$

holds for $l \in I_N$. □

Now we present the necessary condition on the loss probability for stabilization.

Proposition 3.2. *The necessary condition on the packet loss probabilities* α_i *so that there exists a controller that stabilizes the plant is given by*

$$(\prod_{i=1}^{p} \alpha_i^{\frac{n_i}{2}}) \max|\lambda(A)|^N < 1, \tag{6}$$

where $\lambda(\cdot)$ *denotes an eigenvalue.*

Proof. By proposition 3.1, the system is stable iff there exists an N-periodic $P_k > 0$ such that N inequalities in (5) hold. From (5), for all $l \in I_N$ we have the relation:

$$\alpha_{l,0} A_{l,0}^T P_{l+1} A_{l,0} - P_l < \sum_{i=0}^{1} \alpha_{l,i} A_{l,i}^T P_{l+1} A_{l,i} - P_l < 0.$$

The necessary condition for the system to be stable is that $\exists P_k > 0$ such that:

$$\alpha_{l,0} A_{l,0}^T P_{l+1} A_{l,0} - P_l < 0 \text{ for } l \in I_N.$$

Straightforward calculation leads us to the inequality

$$\alpha_1^{n_1} \alpha_2^{n_2} \ldots \alpha_p^{n_p} (A^T)^N P_0 A^N - P_0 < 0, \; P_0 > 0.$$

The solution P_0 exists iff $\alpha_1^{\frac{n_1}{2}} \alpha_2^{\frac{n_2}{2}} \ldots \alpha_p^{\frac{n_p}{2}} A^N$ is a stable matrix i.e $(\prod_{i=1}^{p} \alpha_i^{\frac{n_i}{2}}) \max|\lambda(A)|^N < 1$. □

For $p = 1$ i.e. there is only one virtual channel, the necessary condition of the packet loss probability is given by:

$$\alpha_1 < \frac{1}{\max|\lambda(A)|^2}$$

which is the same as the result of Elia [4)] where the author derived the allowable loss probability for the stochastic stability.

Now assume $p = 2$ with packet loss probability $\alpha_1 = \alpha$ and $\alpha_2 = 1$ (100% packet loss) and $q := [1, ..., 2, ...]$ where we use each channel n_1 and n_2 times respectively. The necessary condition on α is

$$\alpha < \frac{1}{\max|\lambda(A)|^{\frac{2N}{n_1}}}$$

which is similar to the result of Ishii [5)] for SISO plant.

4 Performance of Periodic Switching

In this section we study the performance of periodic switching scheme for two different scenarios.

4.1 Performance Guaranteed Decay Rate

Assume $p = 2$ with loss probability $\alpha_1 = 0$ and $\alpha_2 = \alpha$. We want to determine the minimum data rate over a non-dropping channel or the largest period of length N to satisfy a desired deterministic convergence rate i.e.

$$\sup(\|x_k\|^2) \le c^{-k} \|x_0\|^2 \tag{7}$$

holds for any realization of α_2. First, assume $q := [1, 2, ..., 2]$ ($q := [2, ..., 2, 1]$ will lead to the similar result). The probability that all θ_k in one period are 0 is given by α_2^{N-1} ($\neq 0$). To guarantee the decay rate, consider the worst case where all θ_k in one period are 0 (assuming $\alpha_2 = 1$). Then, at time step N, the state can be written as:

$$x_N = A^{N-1} A_c x_0,$$

where $A_c = A + BK$. Substituting this equality to (7) leads us to :

$$\|A^{N-1} A_c x_0\|^2 \le c^{-N} \|x_0\|^2.$$

Solving this inequality will give us the following result.

Corollary 1. *For* $p = 2$ *and* $\alpha_1 = 0, \alpha_2 = \alpha$, *the largest period of length* N *so that satisfies (7) is given by*

$$N \le \frac{\log(\frac{\|A\|}{\|A_c\|})}{\log(\sqrt{c}\|A\|)}. \tag{8}$$

The minimum data rate of the non-dropping channel is $\frac{1}{N}$ and does not depend on the stochastic variable α_2 which is reasonable because we deterministically set the value of α_2 for the worst case.

4.2 Influence of Periodic Switching to State Convergence Rate

Consider a scalar system (we use notation a and a_c for the scalar case) and assume $p = 2$ with packet loss probability α_1 and α_2. First we apply periodic switching scheme with pattern q:=[1,...,2,...] where $n_1 = n_2 = n$ $(N = 2n)$. The expectation of the norm of the state for periodic switching $(\overline{E})$ in one period is given by:

$$\begin{aligned}\overline{E} &= E[x_{2n}^T x_{2n} \mid x_0] \\ &= x_0[(\alpha_1 a^2 + (1-\alpha_1)a_c^2)^n \\ &\quad (\alpha_2 a^2 + (1-\alpha_2)a_c^2)^n]x_0.\end{aligned}$$

Next assume $\alpha_1 = \alpha_2 = \frac{\alpha_1+\alpha_2}{2}$ which means that we only have one channel with mean loss probability. The expectation of the norm of the state for this channel $(\underline{E})$ in one period is given by:

$$\begin{aligned}\underline{E} &= E[x_{2n}^T x_{2n} \mid x_0] \\ &= x_0[(\frac{\alpha_1+\alpha_2}{2})a^2 + (1-(\frac{\alpha_1+\alpha_2}{2})a_c^2)]^{2n}x_0.\end{aligned}$$

Now assume $n = 1$, the difference of $\underline{E}$ and $\overline{E}$ is

$$\begin{aligned}\underline{E} - \overline{E} &= x_0((\alpha_1-\alpha_2)^2 a^4 + [2(\alpha 1 - \alpha 2) + \\ &\quad 2(\alpha_2-\alpha_1) - (\alpha_1-\alpha_2)^2 - (\alpha_2-\alpha_1)^2] \\ &\quad a_c^2 a^2 + (\alpha_1-\alpha_2)^2 a_c^4)x_0 \\ &= x_0(\alpha_1-\alpha_2)^2[a^2 - a_c^2]^2 x_0 > 0. \qquad (9)\end{aligned}$$

Inequality (9) is also satisfied for $n > 1$. Therefore, we can summarize the result as follow.

Corollary 2. *Compare to one channel with mean loss probability scheme, the periodic switching scheme may improve the state convergence rate.*

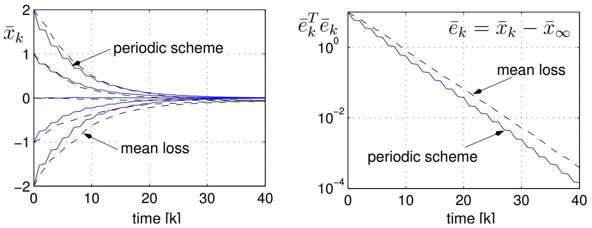
5 Simulation

In this section, we confirm the theoretical results that were presented in the previous section via numerical example. Consider multi-agent systems with collective dynamics in discrete-time given by [7]:

$$x(k+1) = (I - \epsilon L)x(k). \qquad (10)$$

ϵ $(0 < \epsilon < 1/\Delta)$ is the step size, Δ denotes the maximum degree of the network and L is the graph Laplacian. It is shown that for switching network (packet loss probability $(\alpha < 1)$ at all links), consensus is reached if the union of all the graphs is connected [8].

For the simulation it is assumed that there are 5 agents with fully conncected graph. Set $\alpha_1 = 0.1$ and $\alpha_2 = 0.9$. We run simulation for periodic switching scheme with $q := [1, 2]$ and for single channel with mean loss probability of $\frac{\alpha_1+\alpha_2}{2}$. The results for 1000-sample Monte Carlo Simulation are shown in fig. 3. As seen from fig. 3, the state converges faster for the periodic switching scheme. If the network cost increases linearly with the increase of packet loss probability, it means that the state convergence rate may be improved by periodic switching scheme for the same network cost.



(a) Mean value of trajectories of agents

(b) Mean quadratic error norm

Fig. 3: Comparison of Periodic switching and mean loss probability scheme in multi agent systems

6 Conclusion and Future Work

In this paper, we presented QoS control of networked control system by proposing periodic switching scheme of multiple virtual channels with different packet loss probabilities. The necessary and sufficient conditions for stability and necessary condition for the loss probability are derived. We also proved for the scalar case that periodic switching scheme may improve the state convergence rate which is validated by simulation on multi-agent systems. It is left as future work to investigate the optimal switching scheme and extend the model to multi-agent systems.

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