Development and Control of Autonomous, Biped Locomotion using Efficient Modeling, Simulation, and Optimization Techniques

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***Abstract*— Methods for modeling, simulation and optimization of the dynamics, stability and performance of legged robot locomotion are discussed in this paper. It is demonstrated how these tools can assist in the design, implementation and operation of a humanoid robot. The selection and integration of fundamental hard- and software needed for autonomous operation as well as for high agility in biped locomotion are also presented.**

I. INTRODUCTION

The successful development of an autonomous robot must strive for the optimal synthesis of three areas:

i) the functional and physical requirements derived from the envisaged robot application,
ii) the selection and integration of hardware (HW) and software (SW) suited to meet these requirements,
iii) the development and integration of intelligent and efficient algorithms on all levels of control, planning and perception subject to the real-time constraints given by the robot's HW/SW and its task.

Many research groups and companies are developing biped walking machines and placing effort and focus into hardware considerations [1, 3, 5]. The expensive, time-consuming development and production process make it difficult though to compete with private corporations [11–13]. However, towards the development of an effective autonomous robot, all of the three areas mentioned above must be considered. Especially for the development of dynamic biped locomotion, we find it important to model and simulate the biped locomotion dynamics on all levels of the design, implementation and operation phases of a humanoid robot, e.g., for the selection of motors and gears, the development of joint reference trajectories for implementing first steps, the development of nonlinear dynamics model-based locomotion controllers using HW- and SW-in-the-loop environments, for task planning, and for developing autonomous behaviors.

A precise modeling of legged locomotion systems requires high dimensional nonlinear multibody systems (MBS) dynamics with constraints. Further complex tasks are generation, optimization and control of stable motions for such systems. Modeling and simulation can assist in the development of autonomous biped locomotion much more than it is currently being used. This approach complements, but not replaces the selection and integration of HW and SW.

II. EFFICIENT MODELING AND SIMULATION OF DYNAMIC BIPED LOCOMOTION

A. General considerations

Various approaches exist for modeling the MBS dynamics of a tree-structured legged robot subject to unilateral contact constraints, all with quite different characteristics regarding efficiency and accuracy in simulation and optimization. Symbolic methods are required for closed-form dynamic equations which give the best performance in terms of number of arithmetic operations and basic function evaluations needed for evaluation. This approach, though, does not fulfill the need for modularity and flexibility if parts of the kinematical structure or the kinetical data have to be changed and refined as occurs frequently during the design and operation cycle of a humanoid robot. Furthermore, it is desirable to use the same dynamic modeling framework during the entire development and operation period of a legged robot, e.g., for the selection of actuators using dynamic optimization (Sect. IV-A), and for the optimization of reference trajectories for dynamic walking (Sect. III-B), for the calibration of model parameters by optimization, for the model-based estimation of dynamic state variables, and for the future development of nonlinear dynamic model-based controllers realizing dynamically stable legged locomotion. The development of a completely object-oriented C++ toolbox facilitating these investigations is currently underway [10] which will permit the easy interchangeability of actuator, contact, joint, and link models providing further flexibility in these studies. The MBS modeling and computational approach is the Articulated Body Algorithm (ABA) due to its superior modularity and computational efficiency for high dimensional systems [4, 16].

Biped constructions generally consist of a minimum of five bodies with two to six degrees of freedom (DoF) per leg. Dynamical simplifications allow one to analyze certain predominant behaviors of the dynamic system, but many other important features are lost. A more complete dynamical system description contains more significant dynamical effects yet a control solution for these models based on an analytical approach is usually not possible and results must be sought for numerically. The modeling and optimization approaches presented here are thus strongly dependent upon numerical methods.

The basic equations of motion are those for a rigid, multibody system experiencing contact forces

$$\begin{aligned} \ddot{\mathbf{q}} &= \mathcal{M}(\mathbf{q})^{-1}\Big(B\mathbf{u} - \mathcal{C}(\mathbf{q},\dot{\mathbf{q}}) - \mathcal{G}(\mathbf{q}) + J_c(\mathbf{q})^T\mathbf{f}_c\Big) \\ 0 &= \mathbf{g_c}(\mathbf{q}) \end{aligned} \tag{1}$$

where N equals the number of links in the system, $\mathcal{M} \in \mathbb{R}^{N\times N}$ is the square, positive-definite mass-inertia matrix, $\mathcal{C} \in \mathbb{R}^N$ contains the Coriolis and centrifugal forces, $\mathcal{G} \in \mathbb{R}^N$ the gravitational forces, and $\mathbf{u}(t) \in \mathbb{R}^m$ are the control input functions which are mapped with the constant matrix $B \in \mathbb{R}^{N\times m}$ to the actively controlled joints. The ground contact constraints $\mathbf{g_c} \in \mathbb{R}^{n_c}$ represent holonomic constraints on the system from which the constraint Jacobian may be obtained $J_c = \frac{\partial \mathbf{g_c}}{\partial \mathbf{q}} \in \mathbb{R}^{n_c\times N}$, while $\mathbf{f}_c \in \mathbb{R}^{n_c}$ is the ground constraint force.

A great advantage in legged systems is that their constrained contact legs often have unique inverse kinematic solutions for their joint angles and angle velocities. This lends itself to the use of reduced dynamics algorithms for simulation and optimization. This approach, also known as coordinate partitioning [2], projects the dynamics (1) onto a reduced set of independent states thus converting the DAE contact system (1) into an ODE system of minimal size. Using a recursive multibody algorithmic approach, the reduced dynamics may be evaluated without explicitly constructing them [8]. The constant mapping $Z \in \mathbb{R}^{(N-n_c)\times N}$ from the full state vector $\mathbf{q}$ to the independent states $\mathbf{q_I}$ then reduces the dimension of the equations of motion and the system evolution follows automatically the contact manifold. The dependent states $\mathbf{q_D}$ can be calculated from $\mathbf{q_I}$. A partition of the states as $\mathbf{q} = (\mathbf{q_I}, \mathbf{q_D})$ exists that

$$\ddot{\mathbf{q}}_\mathbf{I} = Z\mathcal{M}(\mathbf{q})^{-1}\Big(B\mathbf{u} - \mathcal{C}(\mathbf{q},\dot{\mathbf{q}}) - \mathcal{G}(\mathbf{q}) + J_c^T\mathbf{f}_c\Big) \tag{2}$$

holds. The principal advantage of this approach is that one needs only perform the optimization on the reduced dimensional state. The state must then be monitored such that it remain within a well-defined region of the state space.

An important aspect of formulating a gait optimization problem is establishing the many constraints on the problem. For a biped, the gait cycle consists of several phases describing different contact situtations and being separated by events. The order of contact events is straightforward and depends primarily upon the speed of locomotion. A summary of the modeling constraints for a *complete* gait cycle is [9]:

Periodic gait constraints (gait optimization):

1) Periodicity of continuous state and control variables.
2) Periodicity of ground contact forces.

Exterior environmental constraints:

1) Kinematic constraints on the height (z-coordinate) of the swing leg tips.
2) Ground contact forces lie within the friction cone and unilateral contact constraints are not violated.

Interior modeling constraints:

1) Jump conditions in the system velocities due to inelastic collisions of the legs with the ground.
2) Magnitude bounds on states, controls and control rates.
3) Actuator torque-speed limitations.

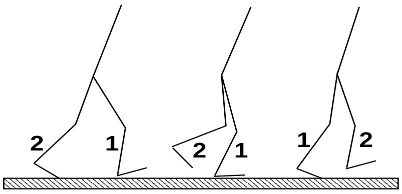

Fig. 1. Three Phases of Dynamic Gait with Different Foot Contact Positions for Leg 1: (1) Heel Roll, (2) Flat Contact, (3) Toe Roll

Depending upon whether a statically stable or dynamically stable biped gait is desired, the optimization problem formulation will have different periodicity, symmetry, and kinematic phase boundary constraints depending on the foot contact positions (Fig. 1). The number of phases may also differ. We model the static and dynamically stable gaits as follows.

Statically Stable Gait:
Phase 1: Foot 1 flat contact, Foot 2 swinging freely
Phase 2: Foot 1 flat contact, Foot 2 flat contact

Dynamically Stable Gait:
Phase 1: Foot 1 heel roll contact, Foot 2 toe roll contact
Phase 2: Foot 1 flat contact, Foot 2 swinging freely
Phase 2: Foot 1 toe roll contact, Foot 2 swinging freely

B. Dynamic model of humanoid robot

As an example, we consider our humanoid currently under development (Fig. 2).

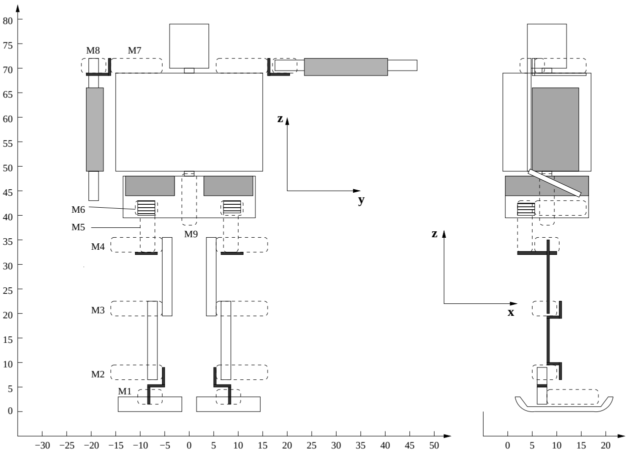

Fig. 2. Humanoid Kinematic Structure

The humanoid construction consists of:

1) two legs each with 6 links and 6 actuated joints
2) hip has 3 DoF, knee 1 DoF, ankle 2 DoF
3) waist joint providing a rotation about vertical axis
4) each shoulder with 2 DoF
5) head is (temporarily) fixed to the body

The humanoid dynamic model consists of:

1) 17 *degrees of freedom*
2) free-floating body with central reference point in the torso and a fictitional 6 DoF joint between it and an inertial reference frame
3) modeled as a tree structured multibody system (contacts are "cut" between robot and ground)

If we restrict motion to the sagittal plane, a minimum set of generalized coordinates consists of 14 position and 14 velocity states $(\mathbf{q}(t), \dot{\mathbf{q}}(t))$, thus a total of 28 first order differential equations. Then the free-floating fictional joint has only 3 DoF and each leg also only 3 DoF. With 3-dimensional motion, there are 23 position and 23 velocity states $(\mathbf{q}(t), \dot{\mathbf{q}}(t))$ resulting in 46 differential equations.

The moment of inertia parameters for the humanoid are estimated in that each individual piece of the construction is approximated by a standard shape: cylinder, ellipsoid, box. These geometrical primitives have a standard inertia tensor assuming a uniform distribution of mass. The masses of the individual elements are weighed by hand.

The ordering of the individual joints is not arbitrary. It was intentional that the hip flexion/extension joint performing most of the work in the hip was placed last of the three hip joints; thus, the needless work of swinging the other two hip joints is saved. On the other hand, the flexion ankle joint is placed higher than the abduction joint so that at collision of the heel with the ground the impulsive force will disperse better throughout the body rather than influence primarily only the ankle joints.

III. DYNAMIC STABILITY AND PERFORMANCE

A. Measures for dynamically stable locomotion

The difficult task of maintaining stability of fast legged locomotion has been a main obstacle in the construction of such systems. The notion of *static stability*, often used to enforce *postural stability* in legged systems, does not suffice for fast motion. Static stability requires the ground projected center of gravity to lie within the *support polygon*, the convex hull about the leg's contact points. This highly conservative measure of postural stability generally results in very slow legged motions. The notion of *dynamic stability* is required for faster legged motion, yet as pointed out in [6], a dynamically stable gait is one without static stability that is sustainable indefinitely. A satisfactory quantitative measure for stability, though, which serves for gait generation and control design is not yet available.

The ZMP is that point on the ground where the total moment generated due to gravity and inertia equals zero or equivalently the point where the net vertical ground reaction force acts. This point has frequently been used to produce stable locomotion beyond the region of static stability [14]. This measure has many deficiencies when considering fast locomotion; in particular, it provides little stability information during the important rolling action of the feet (see Fig. 1) in fast walking and running. The support polygon may then have a zero or reduced surface area, and the ZMP may lie directly on the support boundary, thus on the border of its pre-defined stability region. Numerical investigations performed in [8] demonstrated that when walking on flat feet, the double contact phase of biped walking has no energetical advantage. Combining this observation and the fact that during approximately 80% of a normal human walking gait a rolling foot action takes place, we can deduce that a dynamic stability measure precisely during foot rolling is necessary for fast locomotion.

A stability measure related to the ZMP providing more information as to the system instability was presented in [6]. This FRI point or foot-rotation indicator coincides with the ZMP during periods of static equilibrium of the foot and otherwise provides information as to the foot's rotational instability as a function of the uncompensated rotational torques of the system acting on the foot. From this measure we may define a postural stability performance criterion:

Stability Performance 1: Average distance in the ground plane between the **FRI** point and the ground projected center of mass **GCoM** normalized by the distance traveled s.

$$\mathbf{J}_{s1}[\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u}] = \frac{1}{s} \int_0^{t_f} ||\mathrm{GCoM} - \mathrm{FRI}||^2 \, \mathrm{d}t \qquad (3)$$

This value alone is not sufficient to verify or design a dynamically stable control strategy, yet it may be combined with additional dynamic measures of the system such as the angular momentum which can provide a stability assessment during gait optimization, simulation, and on-line control.

Efficiency is secondary in importance to stability in legged systems, but it can also have a strong influence in the successful design of an autonomous biped. A challenge for systems with limited power supply is to combine energy conserving motion with the robust, stability properties discussed previously. It has been witnessed in humans that steady-state forward walking approximates a minimum energy motion according to a dynamical model for the human body [17]. An attempt to reproduce smooth, natural motion should also take these factors into account.

Energy Performance 1: In legged systems where a high torque is generated by a large current in the motor, the primary form of energy loss is called the Joule thermal loss [15]. One may minimize the integral of this value over the gait:

$$\mathbf{J}_{e1}[\mathbf{u}] = \frac{1}{s}\int_0^{t_f} \sum_{i=1}^{N} R_i \left(\frac{u_i}{G_i K_i}\right)^2 \mathrm{d}t \qquad (4)$$

where R_i, G_i, K_i, and u_i are the armature resistance, gear ratio, torque factor, and applied torque for link i respectively, while s is the step length or total distance traveled over one stride.

Energy Performance 2: Another efficiency cost criterion is the specific resistance ϵ as used in [7]. This measures the output power in relation to the mass moved and the velocity attained and is a dimensionless quantity. Its integral over the gait cycle is a normalized form of the kinetic energy

$$\mathbf{J}_{e2}[\dot{\mathbf{q}}, \mathbf{u}] = \int_0^{t_f} \frac{\sum_{i=1}^{N} |u_i \dot{q}_i|}{mgv}\ , \qquad (5)$$

where mg is the weight of the system, $\dot{q}_i$ is the joint i angle velocity and v is the average forward velocity.

The availability of a fully validated dynamic model combined with optimization tools permits one to make conclusive investigations into which stability or efficiency measures are most effective.

B. Optimization of stability and performance indices

Algebraic control strategies for legged systems cannot yet be constructed to handle the high dimension and many modeling constraints present in the locomotion problem. Heuristic control methods, on the other hand, tend to have poor performance with respect to power efficiency and stability. The remaining proven approach is the use of sophisticated numerical optimization schemes which can incorporate the numerous modeling constraints to generate optimal trajectories. The resulting trajectories may later be tracked or used to approximate a feedback controller in the portion of state space of interest. This section describes several performance indices which may be optimized and then presents the optimization approach.

Numerical optimization tools have advanced sufficiently [18] such that the many modeling and stability constraints can be incorporated into the problem formulation together with a relatively complete dynamical model so as to obtain truly realistic energy-efficient, stable and fast motions. The optimization approach is based on a discretization of the control problem in time using direct collocation and its subsequent formulation as a nonlinear programming problem then solved with a sparse sequential quadratic programming algorithm. This approach has already been successfully applied for gait planning in bipeds in two dimensions [8] and for quadrupeds in three dimensions in [9].

The optimization of the stability or energy performance indices subject to the general system dynamics $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t)$ and constraints leads to optimal control problems (here $\mathbf{x} = (\mathbf{q_I}, \dot{\mathbf{q}}_\mathbf{I})$). Their solution delivers optimal open loop trajectories $\mathbf{x}^*(t)$, $\mathbf{u}^*(t)$, $0 \leq t \leq t_f$. The program DIRCOL [18] uses the method of sparse direct collocation and approximates the states $\mathbf{x}$ with spline functions, the controls $\mathbf{u}$ with linear functions, and constant parameters $\mathbf{p}$ on a discrete time grid. The method is equipped to handle the complexities of the walking problem: unknown liftoff times, different ground contact combinations for the legs, discontinuous states at collision times of the legs with the ground, switching dynamics, and actuation limits.

IV. RESULTS FOR AUTONOMOUS BIPED DESIGN AND DYNAMICS OF LOCOMOTION

A. Design considerations of biped walking machine

One is faced with a difficult compromise in the design of an autonomous biped. Maximum agility and speed of locomotion require strong motors and gears. The actuators, though, must be as light as possible for autonomous operation and without extensive power consumption leading to heavy on-board batteries. A strategy for finding a good compromise between these conflicting goals using dynamic optimization was presented in [19]. The resulting architecture of the 80 cm humanoid robot is shown in Fig. 2.

The optimization criterion used was Energy Performance 1 (4) subject to the complete biped dynamics for a rigid body model (1) and maximum input power constraints. The maximum output wattage M_W in the power constraints was first selected to determine the motor class.

$$\max_{t \in [0,t_f], i \in \{1,\ldots,n\}} |\dot{q}_i(t) u_i(t)| \leq M_W \qquad (6)$$

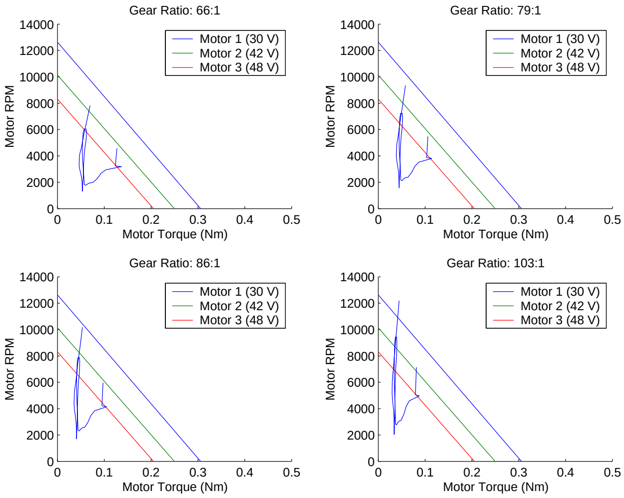

Fig. 3. Motor Torque vs. Rpm from a 18 kg Biped

Consequently, optimal trajectories provide information as to the maximum required torques, joint velocities and accelerations in order to produce a walking gait at a specified forward velocity.

Though commercial high performance motors are available with a wide range of power, torque, and speed output characteristics, A significant void in motor availability generally exists between motors with a maximum power output of $M_W = 20 - 25$W and those with $M_W = 70$W, the latter having a much increased weight. Thus for the investigations, $M_W = 20$W was set for all joints. The optimization problem was solved for various mass configurations such the 18 *kg* model shown in Fig. 3.

Displayed in Fig. 3 are the calculated optimal trajectories in motor torque and rpm space. The required motor torques T_m are calculated from the chosen gear ratio N_i and efficiency hg_i, $T_m = \frac{T}{N_i\, hg_i}$. The motor speed n_r is the gear output speed n_o multiplied by the gear ratio, $n_r = n_o N_i$. Additionally plotted for each given gear ratio are three different motor voltage ratings: $V_m = \{30\text{V}, 42\text{V}, 48\text{V}\}$ assuming a battery supply voltage of $V_s = 38$V delivered by three batteries providing 14.4 V each. The motor characteristic line is calculated by first determining the no-load motor speed n_{mV} from its rated value n_m and adjusted according to the supply voltage V_s, $n_{mV} = n_m \frac{V_s}{V_m}$. The points lying below the motor characteristic line specify the reachable torque and velocity combinations. A necessary condition is thus that the desired motor workspace lie beneath this line. This investigation led to a 42V motor with a 66:1 gear ratio.

B. Selection and integration of fundamental hard- and software

The mechanical biped construction (cf. Fig. 4) is based on linking elementary modules each consisting of motor, gear, pulse encoder, L-shaped base plate, and lever arm [19]. This prototype carries three batteries for the power supply of the motors, two of them visible on the picture at the height of the hips and below the waist joint. The third battery is located symmetrically behind the hips. The chosen Sony BP-L90A batteries provide a capacity of 90 Wh each, hence allowing for approximately 45 min autonomous walking. As a consequence of the design considerations, a refined prototype will be operated with lighter batteries.

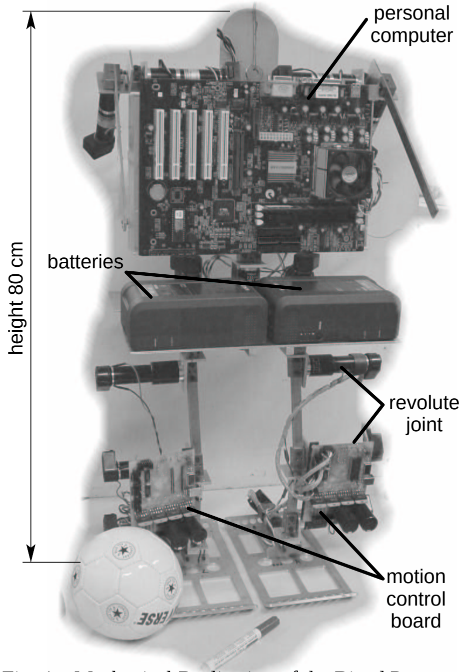

Fig. 4. Mechanical Realization of the Biped Prototype

For this protoype, a standard ATX mainboard with Athlon 1300 MHz CPU has been chosen providing enough computational power for motion control and additional tasks such as object recognition using a camera system. Its power is supplied by two Bebop Endura E-50S batteries.

The motors are accessed using an USB motion control board developed at the Control Systems Group in Berlin [19]. It consists of an 8 bit microcontroller including 3 USB endpoints, a 6 channel A/D converter and a 16 channel pulse-width modulator (PWM) admitting a motor load of up to 3 A at 55 V. The actual motor position is determined by evaluating the signals of pulse encoders attached to each motor. Up to 4 motors can be connected to each board weighing 170 g. In consideration of the USB control transfer mode, mean USB communication delay as well as microcontroller computational times required by USB service routines and PD control routines, PD control loops and communication runs have been designed and implemented at 250 Hz giving satisfactory control performance for this prototype.

Due to the use of microcontroller boards, no hard real-time constraints exist for the main computer where Real-Time Linux has been chosen as the operating system. The communication between on-board PC and the microcontroller boards is triggered by internal Linux timer signals. External communication with the robot is realized by wireless LAN.

The current joint angles are sensed by pulse encoders. The moments acting at each joint may be computed from the sensed motor currents. A gyroscope and an inclination sensor will be mounted in the main body. For collision detection with the ground, the feet should be equipped with contact sensors at the corners. It will be necessary to sense contact forces and moments of the feet with the ground. An interesting question to be investigated is whether the contact forces and moments can be computed from standard internal sensor data using a fully three-dimensional and validated model of the bipedal robot dynamics. Also a stereo camera at the head will be integrated to enable autonomous perception.

A graphical user interface has been developed for rapid control prototyping. The MATLAB Realtime Workshop provides a solution by permitting the generation and compilation of standalone real-time code for RT Linux from a SIMULINK model. The control loop to be implemented is composed of ordinary SIMULINK blocksets, hence the migration from designing a controller in offline mode to evaluating it in an experiment is subject to substituting the system model by hardware in the loop which can also be accessed through SIMULINK blocksets. Another benefit of this procedure is the clear structure compared to a large number of manually linked files of source code. Consequently maintenance and modification of the controller become easier and less time-consuming.

C. First walking experiments for biped prototype

First walking experiments for statically stable leg motions are displayed in Fig. 5 where trajectory following control has been performed with a position error of less than 0.019 rad. The displayed PWM ratio signals are less than 0.5. Thus, the utilized DC motors offer enough power for faster gaits.

Ongoing research is being conducted with dynamic trajectory optimization in three dimensions considering also the foot phases of dynamic gait (Fig. 1) for fast walking applications and their implementation on the prototype. Also the simulation model used for the dynamics of biped locomotion will be validated and refined through systematic comparison of simulation and experiment.

For the final version of the paper, this section will include the most recent experimental and numerical results.

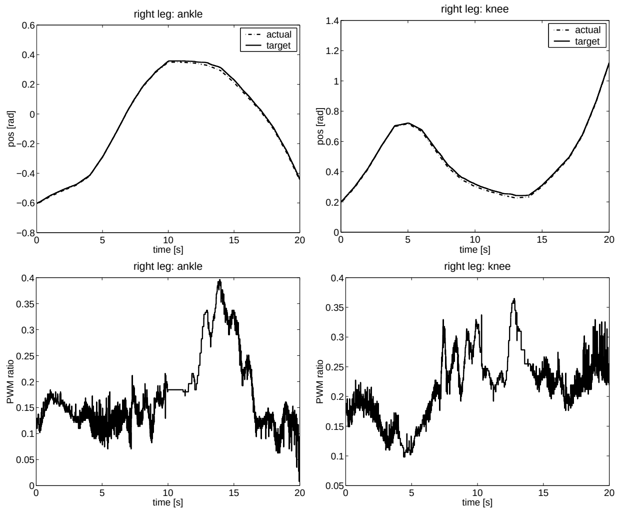

Fig. 5. First Experiments with Trajectory Following Control (left: ankle joint, right: knee joint of right leg)

V. CONCLUSIONS AND OUTLOOK

The development of autonomous bipedal robots, e.g. for robotic soccer, requires results from many incompletely solved research themes including fast locomotion and agility to autonomously react from a quickly changing environment. Modeling, simulation and optimization of complete dynamic models of the legged robot dynamics can greatly assist in making progress towards this goal on all levels of design, implementation and operation. In this paper, efficient approaches for modeling legged robot dynamics as well as measures for stability and performance of legged locomotion have been discussed along with open questions. Results for design considerations and trajectory optimization for an autonomous humanoid robot have been presented. The protoype has been designed towards an overall lightweight system constructed from only a few modular off-the-shelf motors and gears to reduce construction and maintenance efforts. An USB-motion-control-board has been developed facilitating a decentral, microcontroller based joint motor PD control enabling sampling rates of 250 Hz. Onboard energy supply for the motors and the onboard computer is provided by several batteries. First experimental results for statically stable leg motions demonstrate a low position error and the potential of the selected motor-/gear-units for realizing fast gaits. Currently, computations and experiments for fast, three dimensional gaits are underway. The experimental results will also be used to validate and refine the dynamical model of the robot. Further steps will include the development of feedback controllers based on real-time SW-in-the-loop simulations using full multibody dynamical models, motor and gear dynamics as well as models for foot-ground contact and sensors.

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