Infinite Time Coverage Control with Information Decay for Mobile Sensor Networks

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Abstract– In this paper a method for coverage control for a connected and bounded region $\mathcal{D} \subset \mathbb{R}^2$ in a dynamic environment is studied. An information map is introduced in which the information about each point is decaying with respect to time s.t. the robots must revisit them periodically. The considered gradient based control approach causes the cost function to stay within the desired bounds. But because of the non-stationary problem setup with information decay it does not converge to a single point but to a bounded set, so that the robots keep gathering information continuously. With this method it is possible to gather information about several points of interest within the region $\mathcal{D}$ with only a few robots. In the end simulation results are presented to outline the effectiveness of the proposed control law.

Key Words: Coverage control, Information decay, Mobile sensor network

1 Introduction

Natural disasters with their need for quick humanitarian help as well as military and surveillance operations and tasks in hazardous environments are major application for robots. In these kind of applications we often have to deal with additional difficulties like an adversarial environment, constraints (e.g. time restrictions, limited communication capabilities, etc.) and changing mission objectives. But for the use of robots in these scenarios they also must be economically reasonable. This is the motivation for the approach of this paper to use only a few robots, which should gather a information in a specified area.
Recently a lot of research results came up for the area of coverage control. Most of the results rely on Voronoi tessellations. For example in [1)] a gradient descent to reach the optimal Voronoi configuration is proposed and in [2)] coverage control algorithms for robot groups with limited-range interactions are presented. Other results use some explicit information measures to express a gain of information. For instance [3)] proposes an algorithm which causes mobile agents with a dynamic network topology to improve their estimation of a moving target. In [4)] motion coordination algorithms which maximize the determinant of a Fisher Information Matrix are presented and in [7)] a solution to an active sensing task is given which minimizes the variance of the estimation error and thus reduces the uncertainty of the target state. In addition there are also results like [5)] which are based on receding horizon control or MPC.
In this paper the problem of gathering information and monitoring an area with only a few robots is addressed. The aim was to derive results, which are applicable for infinite time and therefore not converge to a single point but to a bounded set. In order to achieve that, an idea similar to the effective coverage function in [6)] but with the novel concept of an information model with information decay is introduced.
The paper is organized as follows. In section 2 the problem setup and the necessary definitions are presented and explained. Then the control law will be introduced and discussed in section 3. The results were validated by extensive simulations. In section 4 such a simulation for a non convex area $\mathcal{D}$ is presented before a conclusion and the possible extensions of this work are stated in section 5.

2 Problem setup

In this paper $\mathcal{D}$ denotes the area which should be monitored by the agents. $\mathcal{D}$ must be a connected and bounded subset of $\mathbb{R}^2$. Let $Q = \mathbb{R}^2$ be the configuration space of the agents.

Density function: Let $\phi(q) : \mathcal{D} \to \mathbb{R}^+$ be the positive semi-definite density function which represents the regions of interest in the area. $\mathbb{R}^+ = \{a \in \mathbb{R} : a \geq 0\}$ will be used throughout the paper. The robots will always spend particular interest on gathering information in regions where the density function has a high value and will gather information about adjacent regions only if they already have gathered enough information about that areas.

Agent model: Let $\mathcal{A} = \{\mathcal{A}_i \mid i \in S = \{1, 2, 3, \ldots, N\}\}$ be the set of agents consisting of the single agents $\mathcal{A}_i$ with N being the number of agents and S being the index set of the fully connected network which means that it contains the indices of all agents. Let $\mathbf{q}_i \in Q$ denote the position of agent $\mathcal{A}_i$. All agents $\mathcal{A}_i$ satisfy the kinematic equation

$$\dot{\mathbf{q}}_i = \mathbf{u}_i, \quad i \in S \tag{1}$$

with $\mathbf{u}_i \in \mathbb{R}^2$ as the control input of agent $\mathcal{A}_i$. It is assumed that the underlaying dynamics of the agents are controlled by low level controllers which use $\mathbf{u}_i$ as reference input.

Measurement function: The measurement function

$$\mathcal{M}_i(s_i) : \mathcal{D} \times Q \to \mathbb{R}^+ \text{ with } s_i = \|\mathbf{p} - \mathbf{q}_i\|^2 \tag{2}$$

of the agent $\mathcal{A}_i$ is defined as a $\mathcal{C}^1$-continuous map that describes the sensing performance of that agent. Sensing performance in this case means how much information the agent can get about a fixed point $\mathbf{p} \in \mathcal{D}$ depending on his own position. Naturally sensing performance is decreasing when the distance to the agent increases, which leads to the assumption of $\mathcal{M}_i$ being a monotonically decreasing function of the distance between a point $\mathbf{p} \in \mathcal{D}$ and the position of the robot $\mathbf{q}_i$: $\left.\frac{\partial \mathcal{M}_i}{\partial s_i}\right|_{s_i=\|\mathbf{p}-\mathbf{q}_i\|^2} \leq 0 \ \forall \mathbf{p} \in \mathcal{D}$ and $\mathbf{q}_i \in Q \quad \forall i \in S$. Additionally the agent can only gather information in a certain area around his

position. Therefore the sensor area of the agent $\mathcal{A}_i$ is defined as $\mathcal{W}_i = \{\mathbf{p} \in \mathcal{D} | \, \|\mathbf{p} - \mathbf{q}_i\| \leq r_i^2\}$ with the sensor range r_i. This leads to the following additional assumption on the measurement function: $\mathcal{M}_i(s_i) = \frac{\partial \mathcal{M}_i}{\partial s_i}\Big|_{s_i = \|\mathbf{p}-\mathbf{q}_i\|^2} = 0 \quad \forall \mathbf{p} \in \mathcal{D} \setminus \mathcal{W}_i = \{\mathbf{p} : s_i \geq r_i^2\}$.
Now let the measurement map

$$\mathcal{M}(\mathbf{s}) = \sum_i \mathcal{M}_i(s_i) \; \forall i \in S \tag{3}$$

be defined as the sum of all measurement functions and $\mathbf{s}$ be the set containing all s_i.
An example of such a measurement function is

$$\mathcal{M}_i(s_i) = \begin{cases} \frac{C_i}{r_i^4}(s_i - r_i^2)^2 & \text{if } s_i \leq r_i \\ 0 & \text{if } s_i > r_i \end{cases} \tag{4}$$

Information model: Let the information map be defined as $I(\mathbf{p}, t) : \mathcal{D} \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$. The evolution of the map is modeled with the following partial differential equation:

$$\frac{\partial}{\partial t} I(\mathbf{p}, t) = \delta \, I(\mathbf{p}, t) + \mathcal{M}(\mathbf{s}) \tag{5}$$

with the decay rate $\delta \leq 0$. The information map indicates how much information the agents have gathered about the points $\mathbf{p} \in \mathcal{D}$. For $\delta < 0$ this information is decaying with time so that all the points have to be revisited frequently in order to keep the information level high. For $\delta = 0$ the results of [6)] are a special case of the results presented in this paper.

Cost function: Let

$$J(t) = \int_{\mathcal{D}} h(I_{ref}(\mathbf{p}) - I(\mathbf{p}, t)) \, \phi(\mathbf{p}) \, d\mathbf{p} \tag{6}$$

be the cost function which should be minimized with the penalty function $h(e_I)$ which penalizes a lack of coverage which in addition is weighted according to the importance of the area denoted by the density function. There are some necessary assumptions on the penalty function $h(e_I)$ with $e_I = I_{ref}(\mathbf{p}) - I(\mathbf{p}, t)$:

A1 $h(e_I)$ must be piecewise $\in \mathcal{C}^1$

A2 $h(e_I), \frac{\partial h}{\partial e_I}(e_I), \frac{\partial^2 h}{\partial e_I^2}(e_I) > 0, \quad \forall e_I > 0$

A3 $h(e_I), \frac{\partial h}{\partial e_I}(e_I), \frac{\partial^2 h}{\partial e_I^2}(e_I) = 0, \quad \forall e_I \leq 0$

For instance the function $h(x) = (max(0, e_I))^2$ will be used for the simulation.
$I_{ref}(\mathbf{p})$ is the reference information map which gives the level of information the robots should gather for each point of $\mathcal{D}$.

3 Main Result

Now the control law is introduced which will minimize the objective function (6) and its validity is proven by a Lyapunov function approach. In addition there will be a short physical interpretation of the problem and a relation between the decay rate δ and the achieved performance will be discovered. Then the theorem will be extended for partially or even non connected robot groups.
The following abbreviations will be used:

$$h'(e_I) = \frac{\partial h}{\partial e_I}\Big|_{e_I = I_{ref}(\mathbf{p}) - I(\mathbf{p},t)}$$
$$h''(e_I) = \frac{\partial^2 h}{\partial e_I^2}\Big|_{e_I = I_{ref}(\mathbf{p}) - I(\mathbf{p},t)}$$
$$\mathcal{M}'(s_i) = \frac{\partial \mathcal{M}_i}{\partial s_i}\Big|_{s_i = \|\mathbf{p}-\mathbf{q}_i\|^2}$$

Fully connected robot group: As a starting point a fully connected robot group is considered, which means that each agent can communicate with all other agents. Thus the index set is $S = \{1, 2, 3, \ldots, N\}$. Consider the following control law

$$\mathbf{u}_i(\mathbf{q}_i) = -k_i \int_{\mathcal{W}_i} h'(e_I) \, \mathcal{M}'(s_i) \, (\mathbf{p} - \mathbf{q}_i) \, \phi(\mathbf{p}) \, d\mathbf{p} \tag{7}$$

with the feedback gains $k_i \in \mathbb{R}^+$.
Please note that the input for each agent $\mathcal{A}_i$ only depends on his own position $\mathbf{q}_i$ explicitly. The information about the positions of the other robots are only used indirectly through the use of the information map $I(\mathbf{p}, t)$ in the penalty function. According to equation (5) only for the evolution of the information map the information about all robot positions is required. This corresponds to the concept of a 'world model' which can often be found in computer science literature on artificial intelligence.
A closer look at the control law will reveal that it 'sums up' the weighted vectors from the robot to each point $\mathbf{p} \in \mathcal{W}_i$. The use of the gradient of the error distribution inside $\mathcal{W}_i$ causes the robot to move in the direction with the maximum error. Through the following remark this can be seen easily.

Remark 1. *Here some properties which are often used in mechanics but also hold for more general problems should be reviewed. For a region $V \subset \mathbb{R}^n$ and a generalized mass density function $\rho(p)$ with $p \in V$, the generalized mass and center of mass is given as follows:*

$$M_V = \int_V \rho(p) \, dp \tag{8}$$
$$C_V = \frac{1}{M_V} \int_V p \, \rho(p) \, dp. \tag{9}$$

By splitting up the integral and using (8) and (9) with $V = \mathcal{W}_i$ and $\rho(p) = h'(e_I)\mathcal{M}'\phi(\mathbf{p})$ we find out that the control law (7) can also be expressed as follows:

$$\mathbf{u}_i(\mathbf{q}_i) = -k_i \, M_{\mathcal{W}_i} \, (C_{\mathcal{W}_i} - \mathbf{q}_i) \tag{10}$$

Note that the control law can become zero if $h'(e_I)$ is equal to zero everywhere in $\mathcal{W}_i$ even if J(t) is not equal to zero. This happens iff the following condition holds:

$$I \geq I_{ref} \; \forall \mathbf{p} \in \mathcal{W}_i \overset{A3}{\Rightarrow} h' = 0 \; \forall \mathbf{p} \in \mathcal{W}_i \overset{(7)}{\Rightarrow} \mathbf{u}_i(\mathbf{q}_i) = 0 \tag{11}$$

To move the robot away from condition (11) the following simple linear control law is used

$$\hat{\mathbf{u}}_i(\mathbf{q}_i) = -\hat{k}_i \, (\mathbf{q}_i - \hat{\mathbf{p}}_i) \tag{12}$$

with the control gains $\hat{k}_i \in \mathbb{R}^+$. The only requirement on the point $\hat{\mathbf{p}}_i$ is that it has to drive the robots away from condition (11). How to choose $\hat{\mathbf{p}}_i$ specifically is not considered in this paper but a possibility is to choose $\hat{\mathbf{p}}_i$ as the nearest point for which condition (11) does not hold.

Theorem 1. *Under the given assumptions the control law*

$$\mathbf{u}_i^*(\mathbf{q}_i) = \begin{cases} \mathbf{u}_i(\mathbf{q}_i) & \text{if} \quad h' \neq 0 \text{ for some } \mathbf{p} \in \mathcal{W}_i \\ \hat{\mathbf{u}}_i(\mathbf{q}_i) & \text{if} \quad h' = 0 \ \forall \mathbf{p} \in \mathcal{W}_i \end{cases} \tag{13}$$

with sufficiently large gains k_i, $\hat{k}_i \in \mathbb{R}^+$ will hold the objective function within the bounds $\int_{\mathcal{D}} h(I_{ref})\phi(\mathbf{p})d\mathbf{p} > J(t) \geq 0 \ \forall t > 0$. Therefore it will cause the robots to continuously gather information in $\mathcal{D}$.

Proof. For the proof (6) will be used as a common Lyapunov function $V(t)$ for the switching control law (13). Due to the non-stationary problem setup it is only possible to keep the cost function within the stated bounds. Therefore the control law is used to ensure $\ddot{V}(t) < 0$ whenever $\dot{V}(t) > 0$ holds. As stated in (7) it is sufficient to integrate the control law only over $\mathcal{W}_i$ but in order to unify the integration process it is also feasible to integrate over $\mathcal{D}$ because $\mathcal{M}'(s_i)$ equals zero outside of $\mathcal{W}_i$. But first the stated bounds should be derived. From (5) it is obvious that the lower bound $I_{min}(\mathbf{p}) = 0 \ \forall \mathbf{p} \in \mathcal{D}$ for $I(\mathbf{p}, t)$ exists. So the maximum difference and because of A2 therefore also the maximum value of the penalty function $h(e_I)$ is $e_I = I_{ref} - I_{min} = I_{ref}$ because according to A3 differences in the other direction ($e_I < 0$) are not penalized. Hence the upper bound for the cost function is $J(t) \leq \int_{\mathcal{D}} h(I_{ref})\phi(\mathbf{p})d\mathbf{p}$. According to A3 the lower bound is $J(t) \geq 0$ for $e_I \leq 0$. But as long as there is at least one robot in $\mathcal{D}$ it will cause the measurement map to be $\mathcal{M}(\mathbf{s}) > 0$ for some $\bar{\mathbf{p}} \in \mathcal{D}$ and according to (5) this will cause the information map to be $I(\bar{\mathbf{p}}, t) > 0$. Hence the cost function will become smaller. So for $N > 0$ it is only possible to reach the upper bound of the cost function with the initial values but not for any $t > 0$. For reaching the lower bound there are two possibilities. First, there must be enough robots s.t. $\mathcal{M}(\mathbf{s}) \geq -\delta \ I_{ref} \ \forall \mathbf{p} \in \mathcal{D}$ holds. Second, for $\delta = 0$ the lower bound will be reached after some time.
The common Lyapunov function and its derivatives will be discussed in the following.

$$V(t) = \int_{\mathcal{D}} \underbrace{h(e_I)}_{\geq 0} \underbrace{\phi(\mathbf{p})}_{\geq 0} d\mathbf{p} \geq 0 \tag{14}$$

$$\dot{V}(t) = \int_{\mathcal{D}} \underbrace{h'(e_I)}_{\geq 0} (\underbrace{-\delta \ I(\mathbf{p}, t)}_{\geq 0} \underbrace{-\mathcal{M}(\mathbf{s})}_{<0}) \underbrace{\phi(\mathbf{p})}_{\geq 0} d\mathbf{p} \overset{!}{\leq} 0 \tag{15}$$

$$\ddot{V}(t) = \int_{\mathcal{D}} \underbrace{h''(e_I) \left(-\delta \ I(\mathbf{p}, t) - \mathcal{M}(\mathbf{s})\right)^2 \phi(\mathbf{p})}_{\geq 0} d\mathbf{p} \tag{16}$$

$$+ \int_{\mathcal{D}} \underbrace{\delta}_{\leq 0} \underbrace{h'(e_I) \left(-\delta \ I(\mathbf{p}, t) - \mathcal{M}(\mathbf{s})\right) \phi(\mathbf{p})}_{\geq 0} d\mathbf{p}$$
$$+ 2 \sum_{i \in S} \underbrace{M_{\mathcal{W}_i}}_{\leq 0} \underbrace{(C_{\mathcal{W}_i} - \mathbf{q}_i) \, \mathbf{u}_i}_{\overset{!}{\geq} 0} \overset{!}{\leq} 0 \tag{17}$$

¿From A2 and A3 it is obvious that inequality (14) is fulfilled with equality iff $e_I = I_{ref}(\mathbf{p}) - I(\mathbf{p}, t) \leq 0 \quad \forall \mathbf{p} \in \mathcal{D}$ holds.
But inequality (15) does not necessarily hold. It can be reformulated as follows such that it shows us bounds on δ.

$$(0 \geq) \ \delta \geq - \frac{\int_{\mathcal{D}} h'(e_I) \ \mathcal{M}(\mathbf{s}) \ \phi(\mathbf{p}, t) \ d\mathbf{p}}{\int_{\mathcal{D}} h'(e_I) \ I(\mathbf{p}, t) \ \phi(\mathbf{p}, t) \ d\mathbf{p}} \tag{18}$$

If this inequality is violated the objective function starts rising. So the goal is to keep the cost function within bounds around that critical point. To achieve that, it has to be verified that it decreases again after a certain time. Therefore inequality (17) has to hold in that case. The indicated signs of the terms in (17) are also only necessarily true in that case. Please note that $(-\delta I(\mathbf{p}, t) - \mathcal{M}(\mathbf{p}, t)) > 0$ does not necessary hold pointwise but should indicate the sign of the integration result. The input $\mathbf{u}_i$ must be chosen s.t. the inequality holds. This can only be achieved with a sufficiently big positive input $\mathbf{u}_i$ as indicated in (17).
But first let $\tilde{S}$ be defined as the arbitrary index set that contains those agents, which use the control law $\mathbf{u}_i$, and $\hat{S}$ as the arbitrary index set that contains those agents, which use $\hat{\mathbf{u}}_i$. Of course $\tilde{S} \bigcup \hat{S} = S$ holds. With the use of (13) and $\mathbf{v}_i = C_{\mathcal{W}i} - \mathbf{q}_i$ the last term of (17) becomes:

$$2 \sum_{i \in S} M_{\mathcal{W}i} \ \mathbf{v}_i \ \mathbf{u}_i = -2 \sum_{i \in \tilde{S}} k_i \ M_{\mathcal{W}i}^2 \ \mathbf{v}_i^2$$
$$- 2 \sum_{j \in \hat{S}} \hat{k}_j \ M_{\mathcal{W}j} \ \mathbf{v}_j \ \left(\mathbf{q}_j - \hat{\mathbf{p}}_j\right)$$

Please note that for all agents $\mathcal{A}_j$ with $j \in \hat{S}$ condition (11) holds and thus because of

$$h'(e_I) = 0 \overset{(8)}{\Rightarrow} M_{\mathcal{W}_j} = 0 \quad \forall p \in \mathcal{W}_j \text{ with } j \in \hat{S} \tag{19}$$

the last term of the equation above is equal to zero so that we obtain:

$$\ddot{V} = \int_{\mathcal{D}} h''(e_I) \ \left(-\delta \ I(\mathbf{p}, t) - \mathcal{M}(\mathbf{s})\right)^2 \ \phi(\mathbf{p}, t) \ d\mathbf{p}$$
$$+ \int_{\mathcal{D}} \delta \ h'(e_I) \ \left(-\delta \ I(\mathbf{p}, t) - \mathcal{M}(\mathbf{s})\right) \ \phi(\mathbf{p}, t) \ d\mathbf{p}$$
$$- 2 \sum_{i \in \tilde{S}} k_i \ M_{\mathcal{W}i}^2 \ \left(C_{\mathcal{D}i} - \mathbf{q}_i\right)^2 \leq 0$$

for k_i sufficiently big. Thus the cost function will stay within the desired bounds. ■

Remark 2 (Discussion of control law $\hat{\mathbf{u}}_i(\mathbf{q}_i)$:)**.** *As obvious from the proof, the switching to the second control mode $\hat{\mathbf{u}}_i(\mathbf{q}_i)$ does not change the stability argument. It is immediately clear (by applying simple linear analysis) that this control law will drive the agents*

$\mathcal{A}_j$ to the point $\hat{\mathbf{p}}_j$ in infinite time. Though, it is not necessary for the robot to reach that point because as soon as there is a point in his sensory range for which condition (11) does not hold (e.g. $\hat{\mathbf{p}}_j$ enters $\mathcal{W}_j$) it will switch back to the first control law. This shows that (12) drives the agents $\mathcal{A}_j$ to a state where (10) will be used again.

Partially connected robot group: Now the validity of theorem 1 for partially or even non connected robot fleets will be shown. But first a worst case scenario is discussed. Theorem 1 also holds for an index set $S_1 = 1$. This means that there is only one single agent $\mathcal{A}_1$ available for covering the area $\mathcal{D}$. Thus theorem 1 holds obviously for a group of disconnected robots. In that case each robot tries to cover $\mathcal{D}$ by himself without recognizing the efforts of the others.
Starting from this point the index sets $\hat{S}_i \subseteq S$ with $i = 1, \ldots, n$ can be defined for n disconnected subgroups with the properties $\bigcup_{i=1}^{n} \hat{S}_i = S$ and $\hat{S}_i \bigcap_{\substack{i,j \\ i \neq j}} \hat{S}_j = \emptyset$.
It is assumed that the robots of each group can communicate with their group members but not with robots of other groups. Now a group specific objective function $\hat{J}_i$ for each group can be defined as

$$\hat{J}_i(t) = \int_{\mathcal{D}} h(I_{ref}(\mathbf{p}) - \hat{I}_i(\mathbf{p}, t)) \; \phi(\mathbf{p}, t) \; d\mathbf{p} \qquad (20)$$

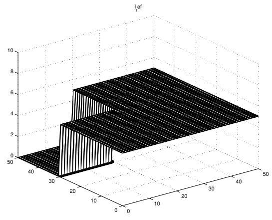
with the group specific information map $\hat{I}_i(\mathbf{p}, t)$, which also evolves according to equation (5) but with a group specific measurement map $\hat{\mathcal{M}}_i(s_i) = \sum_{j \in \hat{S}_i} \mathcal{M}_j$. Obviously the inequality $\hat{J}_i(t) \geq J(t)$ with equality iff $\hat{S}_i = S$ holds. The new common Lyapunov function can be defined as

$$\hat{V}_i = \hat{J}_i(t) \qquad (21)$$

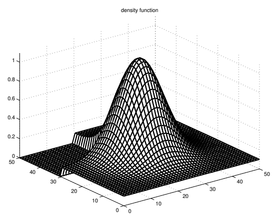
This will lead to similar expressions as we have obtained in the proof for the fully connected robot group and therefore the proof follows exactly the same strategy. This shows us that each robot group for itself tries to cover the points of interest defined by the density function in the area and hence continuously gathers information by moving in the area.

4 Simulation

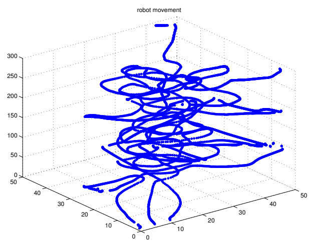
In this chapter we will see 3 robots covering a nonconvex area $\mathcal{D}$. Arbitrary planar areas can easily be represented by simply setting I_{ref} and ϕ to zero in areas which are not accessible for the robots. This will cause the robots to avoid these areas.
The simulation starts with an initial information map $I(\mathbf{p}, 0) = I_0 = 0 \; \forall \mathbf{p} \in \mathcal{D}$. As reference information map $I_{ref}(\mathbf{p}) = 5 \; \forall \mathbf{p} \in \mathcal{D}$ is used and depicted in figure 1(a) and for ϕ the used Gaussian is depicted in figure 1(b). $\mathcal{D}$ and ϕ are embedded in a square region of 50 units length. The measurement function (4) with the sensor range $r_i = 15$ and the peak sensing capacity $C_i = 2$ and the penalty function $h(x) = (max(0, x))^2$ are used. The decay rate and the control gains are set to $\delta = -0.05$ and $k_i = \hat{k}_i = 5$ respectively. For the integration a simple trapezoidal method is applied.
In figure 1(c) the movements of the robots in the x-y-plane is shown with the time in the z-direction and in figure 1(d) the value of the objective function is shown with respect to the time.



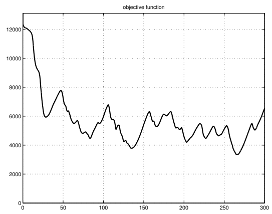
(a) I_{ref}



(b) density function



(c) robot movement



(d) objective function

Fig. 1: Simulation

5 Conclusion

We have formulated a novel concept of a information model with information decay in order to derive a control law that causes small groups of robots to gather information about an area for infinite time. There are three main areas of future work. First, moving targets and collision avoidance should be considered. Second, dynamic network topologies should be introduced and better cooperation might be achieved by using MPC. And third, the extensions to SE(3) and the use of anisotropic sensors might be interesting. Additionally the connection to artificial intelligence noted in section 3 could perhaps be further exploited.

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