Passive and Accurate Torque Control of Series Elastic Actuators

Heike Vallery, Ralf Ekkelenkamp, Herman van der Kooij and Martin Buss

***Abstract*— The principle of Series Elastic Actuation offers considerable advantages for haptic displays compared to stiff actuators. The interaction force between motor and load is directly proportional to their relative position, which corresponds to the elongation of the elastic element. This way, the torque control task is transformed to a position control task, which comes natural to traditional DC motors. In this paper, several existing control strategies are analyzed and compared with respect to passivity concerns. Cascaded control with a fast inner velocity loop results to be the best option. Based on the analysis, boundaries for the parameters are presented, such that the force controller may contain integral action without jeopardizing passivity.**

I. Introduction

For the control of haptic manipulators, which interact with a human being, safety concerns are among the most important issues during the design process. Stability of the manipulator, however, does not suffice. The manipulator and the human, being in direct contact, become a coupled system; and, unfortunatly, coupled systems may become unstable, even if all its components are stable. The main problem is that the control strategies of the human are far from known, complicating the design process of the manipulator. The general procedure is to design the manipulator in such a way that the range of stable couplings with different environments is as large as possible. For the family of passive systems, general statements about coupled stability can be made: It can be shown that passive systems coupled in feedback or in parallel manner again yield a passive system. Therefore, it is comonly assumed that the human being acts as a passive system, resulting in the requirement that the controlled manipulator needs to be passive.

For the actuation of haptic interfaces, a variety of actuation principles is possible, e.g. as surveyed in [1] or [2]. Whereas the design of passive control for stiff actuators has been investigated for a long time, series elastic actuators (”SEA”s) still represent a young actuation principle for haptic manipulators. Pratt proposed such a compliant actuation together with a force control concept in [3]. Before, compliance had always been considered a disturbing component and was treated as such [4]. Although an elastic concept does have several drawbacks, such as the limited bandwith [5] and stability limitations due to the non-collocation of sensors and actuators [6], it has its advantages in the extremely low realizable impedance. As the force-elongation relationship of the elastic element is known, the elongation can be used for a very cheap force sensor. Furthermore, in contrast to stiff actuators, where actuator saturation leads to high torques at high load accelerations (such as the onset of a movement), a series elastic actuator takes on the natural impedance of the elasticity at high frequencies.

A large number of SEA designs has been presented, e.g. by the MIT [7], [8], Sensinger and Weir [9]–[11], or Wyeth [12]. Similar principles have also been investigated, such as e.g. the SDA (Series Damper Actuator) [13], or a compliance in parallel with the actuator [14], [15]. Furthermore, there is a large number of not actuated, but adaptable elastic elements, i.e. the Variable Stiffness Mechanism [16]. SEAs are used for various applications, e.g. for walking robots [17], for prosthetic and orthotic leg systems [18], or for force-sensing robot arms [19].

Force control of Series Elastic Actuators is generally kept very simple and based on linear control. However, in contrast to stability [20], passivity concerns have only rarely been investigated. In [3], a passive control concept is presented that is based on several feedforward compensation terms and a ”PID”-controller. The drawback is that the integral part of the controller is replaced by a first-order lowpass in order to ensure passivity. Thus, this controller does not counteract static errors (e.g. due to friction). Several years later, Pratt presented another control scheme [21], which couples outer impedance loop and inner force control loop. Besides a desired offset torque, the outer impedance loop commands virtual stiffness and damping to the inner loop, such that a structure-varying system results. As stability and passivity issues of such time variant systems are quite tedious, we do not consider this control scheme here. Furthermore, this scheme seems not suitable for impedances close to zero, because nonzero virtual elasticity and damping parameters have to be given.

In this contribution, several control schemes for Series Elastic Actuators are investigated with respect to passivity issues. As a result, cascaded control with inner velocity loop is chosen and passivity-ensuring boundaries for controller parameters are calculated. The presented control design process is straightforward and allows proceding from the inside to the outside. A first application of this torque controller is the gait rehabilitation robot LOPES of the university of Twente [22]. In this treadmill-based exoskeleton, the torques are transmitted from synchronous motors via Bowden cables and springs to the joints. A major challenge for the controller in this setup is the added elasticity and high friction caused by the Bowden cable transmission. Using this testbed, ex-

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perimental results are presented to support the effectivity of the control approach.

II. System Description

In this section, the uncontrolled plant will be described, which consists of a motor coupled to a load via a series elasticity.

In a series elastic actuator, the load is coupled to the motor via a compliant element, in this case a spring with linear characteristic. A relative displacement of load and motor provokes a spring torque M_L. This principle is illustrated in Fig. 1. The differential equation for this system is

$$J_M \ddot{\varphi}_M = M_M - M_L = M_M - K(\varphi_M - \varphi_L), \quad (1)$$

with φ_L and φ_M denoting load and motor angles, respectively, J_M the motor moment of inertia, M_M the motor torque and K the torsion spring constant. Fig. 2 displays the block chart. Seen from the motor, the uncontrolled

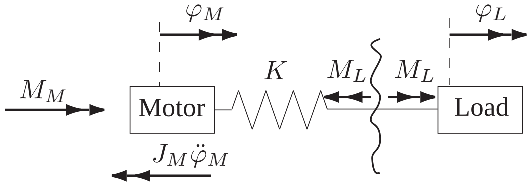

Fig. 1. Series Elastic Actuator: The actuator is connected to the load through a compliant element (a spring). Thus, the spring length is a direct measure of the torque.

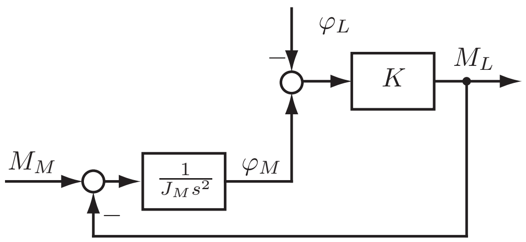

Fig. 2. Block chart of the plant, i.e. the uncontrolled SEA

system ($M_M = 0$) has the input $-\varphi_L$ (load displacement that provokes spring compression), and the output M_L (the spring torque that counteracts compression). Thus, the position-torque transfer function of the uncontrolled plant is:

$$G_p = \frac{M_L}{-\varphi_L} = \frac{K}{1 + \frac{1}{J_M s^2} K} = \frac{J_M K s^2}{J_M s^2 + K}. \quad (2)$$

III. Controller Synthesis

A. Generalization of Existing Strategies

Now, a torque controller is designed to make the spring torque M_L track the desired spring torque $M_{L,d}$. An important constraint is the conservation of stability and passivity aspects, and the goal is to reach a relationship as close as possible to $M_L = M_{L,d}$, which would also allow for an ideal realization of zero impedance. Stability and passivity aspects lead to certain boundaries for the control parameters, and combined with the control goal, a constrained optimization problem for the parameters results.

The controller suggested by Pratt [3] is a simple PID torque controller with some feedback terms added to it:

$$\begin{aligned} M_M &= M_{L,d} + G_{PID}(M_{L,d} - M_L) \\ &\quad + \frac{J_M}{K} s^2 M_{L,d} + K_b J_M s^2 \varphi_L, \end{aligned} \quad (3)$$

with

$$G_{PID} = K_p + \frac{K_d s}{Ts + 1} + \frac{K_i}{s}. \quad (4)$$

If there is no sensor for the acceleration $\ddot{\varphi}$, then the last term cannot be implemented, because filtered differentation will derogate passivity for any value of K_b. Therefore, we neglect it from here on. Furthermore, the system will not be passive for any value $K_i = 0$, such that [3] decided to replace the integrator term by a first order lowpass, which, however, does not counteract static errors. Later, the group proposed driving the motor by a voltage source for the robot Troody [21], this way closing an inner loop for the motor velocity, which is in their case measured via Hall sensors. Wyeth [12] also suggested using such a cascaded control loop with PI controllers, but encoder-based.

Figure 3 shows such a cascaded control with inner velocity loop.

With the desired velocity

$$\omega_{M,d} = G_{PID,o}(M_{L,d} - M_L), \quad (5)$$

the general control law can be written as:

$$M_M = G_{PI,i}[\omega_{M,d} - \frac{s\varphi_M}{T_d s + 1}] \quad (6)$$

$$= G_{PI,i}[G_{PID,o}(M_{L,d} - M_L) - \varphi_M \frac{s}{T_d s + 1}], \quad (7)$$

with the inner and outer controllers

$$G_{PI,i} = K_{pi} + \frac{K_{ii}}{s}, G_{PID,o} = K_{po} + \frac{K_{io}}{s} + \frac{K_{do} s}{Ts + 1}. \quad (8)$$

As the encoder signal is less noisy than the sensor of spring length measurement, the time constant T of the differentiation filter of the spring torque is larger than T_d. If the inner velocity loop is already closed in the PWM with Hall-based velocity sensors instead of filtered differentation of the encoder signal, the inner phase delay vanishes, $T_d \approx 0$.

In contrast to the single-loop force control above, this cascaded scheme can ensure passivity while still counteracting static errors, if some boundaries on the parameters are obeyed. This will be outlined in the following.

The plant output, the spring torque, can be calculated based on superposition of its responses to load displacement and control input to be:

$$M_L = G_1 M_M + G_2 \varphi_L. \quad (9)$$

In haptic systems, the impedance Z is generally defined as the transfer function from input velocity to opposing torque, and it defines the system's passivity. Here, Z is thus defined as

$$Z = \frac{M_L}{-\varphi_L s} = -\frac{G_2}{s}. \quad (10)$$

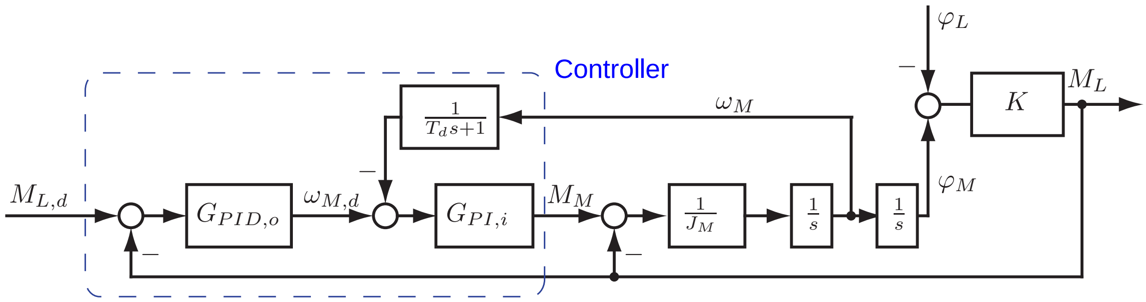

Fig. 3. Torque controller with inner velocity loop. The motor dynamics are represented by the inertia J_M. The spring torque is proportional to the difference between motor angle φ_M and joint angle φ_L, the spring constant is K. A Hall-based velocitiy control in the PWM is a special case of this chart, then no filtered differention has to be considered and $T_d = 0$.

and equals:

$$Z(s) = \frac{K(K_{pi}s + K_{ii} + J_M T_d s^3 + J_M s^2)s(Ts+1)}{\sum_{i=0}^{6} d_i s^i} \tag{11}$$

with

$$\begin{aligned}
d_6 &= J_M T_d T \\
d_5 &= J_M (T_d + T) \\
d_4 &= K(K_{po}K_{pi} + 1)T_d T + K K_{do} K_{pi} T_d + J_M \\
&\quad + T K_{pi} \\
d_3 &= K_{pi} + T K_{ii} + K[(K_{io}K_{pi} + K_{po}K_{ii})T_d T \\
&\quad + (K_{po}K_{pi} + 1)(T_d + T) + K_{do}K_{pi} + K_{do}K_{ii}T_d] \\
d_2 &= K_{ii} + K[(K_{io}K_{pi} + K_{ii}K_{po})(Td + T) \\
&\quad + K_{po}K_{pi} + K_{do}K_{ii} + 1 + K_{io}K_{ii}TT_d] \\
d_1 &= K[K_{ii}K_{io}(T + T_d) + K_{io}K_{pi} + K_{ii}K_{po}] \\
d_0 &= K K_{io} K_{ii}.
\end{aligned}$$

B. *Stability and Passivity of the controlled system*

Necessary and sufficient conditions for passivity of linear systems are [23]:

- $Z(s)$ is asymptotically stable
- $\mathrm{Re}(Z(j\omega)) \geq 0$.

First, the real part of $Z(j\omega)$ is calculated, which gives:

$$\mathrm{Re}(Z(j\omega)) = r(a\omega^8 + b\omega^6 + c\omega^4 + d\omega^2). \tag{12}$$

with

$$\begin{aligned}
a &= K^2 J_M T_d^2 [K_{do}(K_{pi} - T K_{ii}) \\
&\quad - T^2 (K_{io}K_{pi} + K_{po}K_{ii})] \\
b &= K^2 [K_{do}[(K_{pi} - K_{ii}T)J_M + (T - T_d)K_{pi}^2] \\
&\quad + T^2[(K_{pi} - K_{ii}T_d) + K_{pi}^2(K_{io}T_d + K_{po})] \\
&\quad - J_M(T^2 + T_d^2)(K_{io}K_{pi} + K_{po}K_{ii})] \\
c &= K^2[K_{ii}^2(K_{do}(T - T_d) + T^2(K_{po} + K_{io}T_d)) \\
&\quad + K_{pi}^2(K_{io}T_d + K_{po}) - J_M(K_{ii}K_{po} + K_{pi}K_{io}) \\
&\quad + K_{pi} - K_{ii}T_d] \\
d &= K^2 K_{ii}^2 (K_{io}T_d + K_{po}) \\
r &> 0
\end{aligned}$$

Equation (12) is nonnegative for all $\omega \neq 0$, if all coefficients a, b, c, d are nonnegative. Therefore, during the controller design, these inequalities have to be checked. First, the case of encoder-based velocity control is considered, thus a phase delay in the differentiation with $T_d \neq 0$. Then, in order to ensure positiveness of coefficient b, there are only two options: Either the integrators cannot be used, or an outer K_{do} has to be implemented. However, the latter is undesirable in most cases, because it requires numerical differentiation of two possibly noisy signals: the torque sensor signal, and the reference signal, which generally originates from an outer impedance controller. Therefore, T_d should be as small as possible, ideally it should completely be removed by using a motor velocity sensor. However, this is a conservative, theoretical case. In a technical realization, there is always damping present, e.g. due to friction. Viscous friction v, which can be modelled by replacing the term $(J_M s^2)$ by $(J_M s^2 + vs)$, also allows to implement nonzero values for both K_i. However, for the Bowden cables used, friction is hard to quantify due to the complex and highly time variant behaviour.

For easy tuning of both controllers separately, a possible set of simple rules that ensures obedience of the above passivity boundaries is (assuming $T_d < T$):

$$\begin{aligned}
& K_{pi} > J_M \\
\wedge \quad & K_{ii} < 0.5 K_{pi} \\
\wedge \quad & K_{io} < 0.5 K_{po} \\
\wedge \quad & K_{do} > 4T^2 K_{po}.
\end{aligned} \tag{13}$$

By checking the Hurwitz determinants, it can be shown that the conditions above also ensure asymptotic stability. If $T_d = 0$, the inequalities generating from (12) become very simple and K_{do} may be omitted. However, the first 3 rules should still be followed.

C. *Choice of Parameters*

The goal of the controller tuning is on the one hand good tracking of the reference torque, on the other hand good "'disturbance rejection"', i.e. an impedance as low as possible for zero-torque tasks. The transfer function G_F between desired and actual spring torque has the same characteristic polynomial as the impedance transfer function

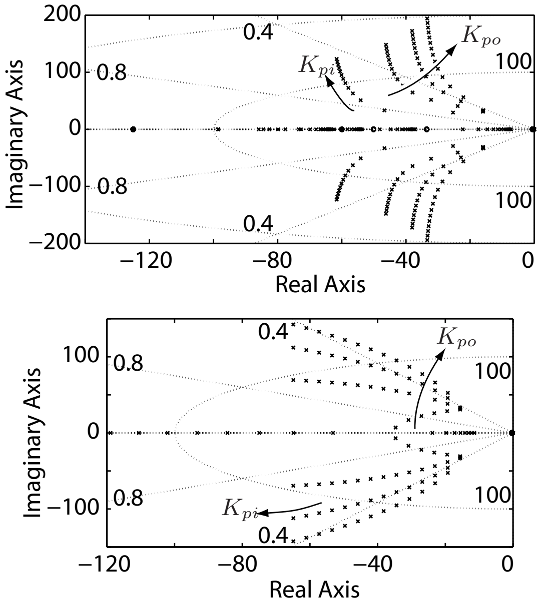

Fig. 4. Root loci of the controlled system with varying K_{pi} and K_{po}. Above: first order lowpass in the velocity signal with $T_d = 0.008$. Below: no phase delay in the velocity control loop. K_{po} varies between 0.01 and 1.5, K_{pi} between 0.1 and 14. The grid lines indicate constant damping ratio and natural frequency, respectively.

(11). Via root locus analysis, the wandering of the poles in dependence of the parameters can be analyzed.

In our technical realization with the LOPES robot, the mechanical properties motor inertia and spring constant are $J_M = 0.131$ kgm^2 and $K = 155$ Nm/rad, respectively. Furthermore, an average measured viscous friction of 4 Nm/(rad/s) is also included in the model.

Fig. 4 displays the root loci for this system using technically realizable values of K_{pi} and K_{po}, both for the case of no phase delay in the velocity signal (interna PWM velocity controller) and with a first-order-lowpass with $T_d = 0.008$ (encoder-based control). The parameters K_i have no significant influence on the dominant poles of the system, especially not on the imaginary poles. They do improve the behaviour at low frequencies though. Furthermore, in a practical realization, integrators can be used to avoid static offsets due to friction. In the simulations, both gains equal 1/3 of the respective proportional gains, in order to safely obey (13).

IV. Practical Evaluation

For the practical evaluation, we use an inner velocity loop in the PWM to control a brushless synchronous motor. The SEA is controlled via MATLAB/Simulink xpc, the sampling frequency is 1000 Hz. For the force controller, we choose control parameters $K_{po} = 0.8$ rad/s/Nm and $K_{pi} =$ 5 Nm/rad/s, with the integrator gains at 1/3 of the respective proportional gains. The joint is restrained manually in the experiments. In Fig. 5, the tracking performance for different frequencies is illustrated. The time plot illustrates the influence of friction, which is compensated quite well. Frequency analysis of the input-output data shows a bandwidth of 16 Hz, with a phase lag of 112° at this cutoff frequency. However, motor saturation prohibits amplitudes larger than 5 Nm at this frequency. The analysis is performed using system identification methods (Matlab procedure: pem). The theoretic response for comparison also includes a measured average viscous friction of 4 Nm/(rad/s). In order to illustrate the zero-torque behaviour, the joint was moved manually at different frequencies. The resulting interaction torques are displayed in Fig. 6. At the onset of a motion, peak torques can be noticed, probably due to backlash and stick-slip. However, these are hardly perceivable in practice, and maximum torques range around 5 Nm.

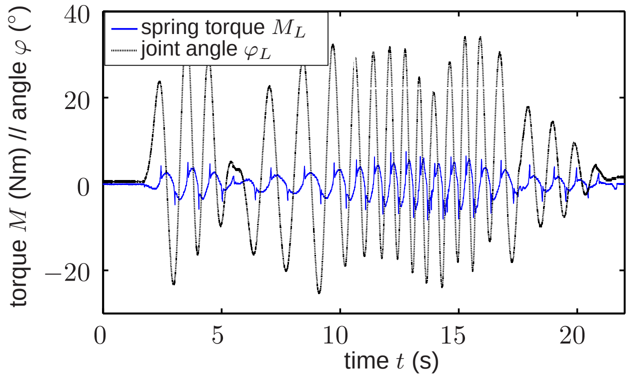

Fig. 6. User-induced joint movement and resulting interaction torque in ”Zero Torque” mode of the controlled SEA.

V. Conclusion

A systematic analysis of existing approaches to torque control of Series Elastic Actuators with focus on passivity has been presented. The resulting recommended control is based on cascaded PI controllers with an inner motor velocity loop. Using stability and passivity analysis, simple boundaries for the control parameters are calculated. The advantages of the scheme are the the possibility to include integral action without jeopardizing passivity. Both theoretically and in practical experiments, the effectiveness of a fast inner velocity loop for good force tracking and low realizable impedances has been shown. Further research will aim at improving the mechanical realization, i.e. the actuation of the LOPES exoskeleton, by removing unwanted elasticity, friction and backlash in the transmission.

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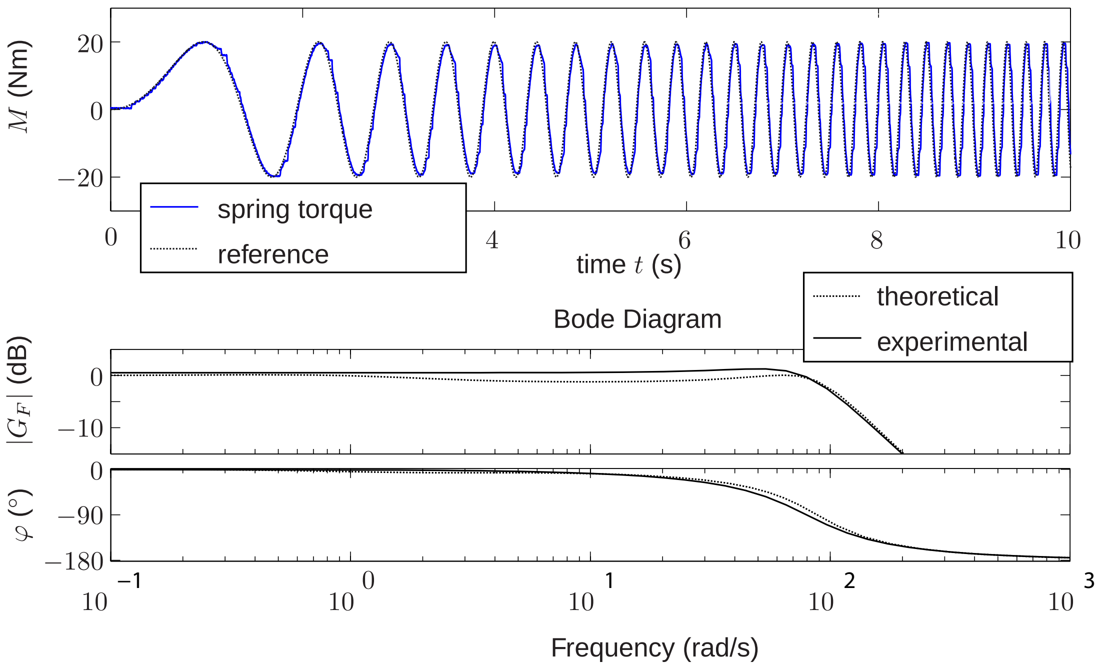

Fig. 5. Tracking performance of the Controlled SEA, assessed via a sine sweep. Above, an excerpt of the time response is plotted, with frequencies ranging from 0 to 6 Hz. Below, the bode plot illustrates magnitude and phase of the experimental and the theoretically expected frequency response.

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