# System Interdependence Analysis For Autonomous Mobile Robots

Florian Rohrmüller, Georgios Lidoris, Dirk Wollherr and Martin Buss

***Abstract*— Autonomous mobile robots are deployed in a variety of application domains, resulting in scenario specific implementations. However these systems share common components responsible for perception, path planning and task execution. In order to find a formal way to identify the influence of the environmental complexity to the used methods, an approach for quantitative system interdependence analysis is introduced. The coherence between several performance indicators of different system components, as well as the influence of environmental parameters on the system, are learned and quantitatively evaluated. Performance evaluation of an autonomous robot navigating in two different urban environments is conducted and presented results demonstrate the applicability of the proposed approach.**

## I. INTRODUCTION

Autonomous robotic systems are called to operate in a wide variety of environments, which all require specific types of capabilities in order to handle the arising complexities. Guide or mall robots [1] are examples of robotic applications in structured indoor environments. While navigation in such situations can be often simplified by providing the robots with complete environment knowledge, still challenges arise through the interaction with people in order to provide them with information. Gradually, the application domain of social robots has been extended to unstructured urban environments [2], while other robots operate in the desert [3], where no interaction is required at all but the focus is primarily on fully autonomous high-speed driving.

All these systems carry out tasks in partially known or unknown environments and are constantly faced with situations that require decision making capabilities under perceptual uncertainty. In order to ensure the robustness of such autonomous robots, it is of high interest to identify the crucial environmental and system component performance indicators and how they influence the overall system behavior. This way the robots can anticipate failures, by predicting the effects that actions would have and correctly adjust their behavior.

Most autonomous mobile robots are complex systems, consisting of several components. Commonly, these components can be separated into three categories according to their purpose. Perceptual components are responsible for building an environment representation e.g. in form of a map and also for localizing the robot. This representation is consequently used to calculate the trajectory of the robot by path planning components. Finally, the chosen trajectory is executed and the progress is monitored by task execution components. It is obvious that sensing, planning and motion execution are interconnected. Although performance indicators have been proposed for each of these components, it is still not possible to assess the effect that environmental parameters, or changes in performance indicators of specific system components, have on the rest of the system.

In order to find a formal way to compare the applicability of methods to these shared problems, the focus of this paper is on the identification of the sensitivity of a robotic system to the changes within its environment. In this respect, performance is in the following expressed as system stability against external influences. Moreover the interdependence between system components, such as perception, planning and execution, is learned, which enables the determination of crucial system components with respect to robustness. This knowledge can be helpful during system development and even be integrated into the on-line reasoning process of the system to enhance its autonomy.

The remainder of this paper is organized as follows: Sec. II summarizes related work followed by a description of the presented approach in Sec. III. In Sec. IV a set of performance indicators is specified. Sec. V shows how Bayesian Networks are learned to represent the coherence between indicators and in Sec. VI information-theoretic criteria are presented to evaluate the degree of the learned coherence. Results based on an autonomous robot navigating in urban environments are presented in Sec. VII.

## II. RELATED WORK

The existence of literature focusing on the performance evaluation of autonomous systems confirms the importance of such methods. Qualitative evaluation criteria of robotic systems have been proposed in [4]. These approaches focus on task objective and social measures to identify both, the efficiency of robot and human. In [5] an evaluation framework for characterizing the autonomy of unmanned vehicles by considering mission complexity, environmental difficulty and HRI is presented. However in order to apply these concepts to embodied autonomous robots and compare their performance with other existing systems and different environments, benchmarks and quantitative performance evaluation criteria are required.

Benchmark scenarios, such as the DARPA Grand Challenge [6] and RoboCup@Home [7], are used to compare the performance of autonomous systems. A similar way to provide reproducibility of environmental conditions is to standardize test arenas for mobile robots [8]. However, such benchmarks do not provide the comparison of robotic systems applied in different scenarios. For example, it is not

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possible to compare a robot which was build to operate in a home environment [9] with autonomous vehicles, which are supposed to navigate through an urban environment [10]. Actually, the scenario-dependence is so strong, that the winning vehicle of the first DARPA Grand Challenge [3] would not be able to take part in the second challenge, since the scenario changed from the desert to an urban environment. Another risk of standardized benchmark scenarios is the resulting intensified development of robotic systems for these specific situations. A problem in this respect might also be the adaptation of algorithms to these specific situations for the cost of generality.

Further approaches introduce quantitative metrics to evaluate robot performance and the influence of the environment on it during navigation missions. Several metrics are proposed in [11] to characterize path quality. The entropy and the compressibility of the environmental information are used in [12] to estimate the complexity of an environment. This method can be also used to identify attractor points. The relation between the environment and the performance of a robotic system is learned in [13] by using a Dynamic Bayesian Network. This way the coherence among the metrics and also the environment is identified. To what extent the performance of one system component influences the quality of another is determined in [14], where the degree of autonomy of a robot is evaluated by combining task performance with world complexity. However, no formal method is presented and the evaluation is based only on simulated data assuming complete knowledge of the environment.

In the approach presented in this paper, several performance indicators for the different system components are discussed and a method for quantitative system interdependence analysis is introduced. The structure of a Bayesian Network is learned from experimental data, in order to identify the coherence between external and internal indicators. Subsequently information-theoretic criteria are used for the evaluation of the coherence. This way, the situation and the effect, that changes in performance indicators of specific system components have on the rest of the system, can be assessed. Results from the system analysis are presented, using data gathered during field experiments with the *ACE* robot [15].

## III. OVERVIEW OF THE PROPOSED APPROACH

In this section a method for system interdependence analysis is proposed. It enables the measurement and evaluation of system performance with respect to the environment parameters and also provides means for a robot to reason about its current state and the interdependences between system components. The approach is illustrated in Fig. 1.

As the robot operates, system outputs are monitored and performance indicators for each of the system components are calculated. Their values are used to learn the structure of a Bayesian Network (BN) off-line and train its parameters. The learned structure identifies the coherence between indicators. In order to quantitatively evaluate this coherence between indicators from different system parts, information-theoretic analysis is performed on the parameters of the learned BN. In the next sections each of the steps is going to be analyzed. The acquired quantitative relation can consequently be used to adjust the on-line functionality of the robot to the situation. This is illustrated by the dashed line in Fig. 1. However, an analysis of how this can be achieved reaches beyond the scope of this paper.

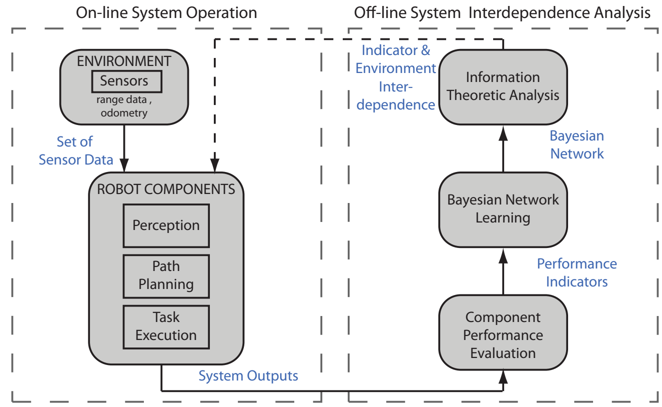


Fig. 1. Flow chart of the proposed system interdependence analysis.

TABLE I
PERFORMANCE INDICATORS

| | |
|---|---|
| Perceptual Ind.: | $H^m$, $H^P$, INFO |
| Planning Ind.: | $s_p$, $n_w$, $\mathrm{var}(\angle(w^1,\phi_r))$, cad, $n_v$ |
| Execution Ind.: | $v_r$, $\mathrm{var}(\phi_r)$ |

## IV. PERFORMANCE INDICATORS

The determination of system performance requires prior identification of adequate indicators. The proposed interdependence analysis approach can handle arbitrary indicators which can be defined by the system designer. However in order to highlight the performance of the approach based on the *ACE* robot, the ten indicators shown in Table I have been derived empirically to describe the internal system state. According to the system architecture in [15], the indicators are grouped into three categories.

*1) Perceptual Indicators:* Perceptual indicators describe the uncertainty of the robot about its position and its environment model. Map uncertainty can be measured by the entropy $H^m$ of the map. For the case of an occupancy grid $m$ this is given by

$$H^m = -r^2 \sum_{l \in m} -p(l)\log p(l) + (1-p(l))\log p(1-p(l)), \quad (1)$$

where $l$ is a cell, $p(l)$ the occupancy probability of $l$ and $r$ the resolution of $m$ [16].

Pose uncertainty

$$H^P = H(p(X_t|Z_t,U_t)) \approx \frac{1}{t}\sum_{j=1}^{t} H(p(x_t|z_t,u_t)), \quad (2)$$

is given as an average over the uncertainty of the different poses along the path as proposed in [17].

Finally, map information $\mathrm{INFO}(m_{t-1}\|m_t)$ is defined as the relative entropy of $m_{t-1}$ with respect to $m_t$, where $m_t$ is the local map at time $t$. The local map $m_t$ is extracted from the occupancy grid $m$, by taking an area $10\,\mathrm{m}\times 10\,\mathrm{m}$ around the robot. $m_{t-1}$ is the spatially corresponding part of the respective map at the previous time step. The relative entropy

$$D_l(p_{t-1}(l)\|p_t(l)) = p_{t-1}(l)\times\log\frac{p_{t-1}(l)}{p_t(l)}, \tag{3}$$

for cell $l$ is also known as Kullback-Leibler divergence. By taking the sum of the symmetric form

$$\mathrm{info}_l(m_{t-1}\|m_t) = \frac{D_l(p_{t-1}(l)\|p_t(l)) + D_l(p_t(l)\|p_{t-1}(l))}{2}, \tag{4}$$

the relative quantity of information around the robot

$$\mathrm{INFO}(m_{t-1}\|m_t) = \frac{1}{N}\sum_{l\in m_t}\mathrm{info}_l(m_{t-1}\|m_t) \tag{5}$$

is derived similar to [13], where $N$ is a normalization factor.

*2) Planning Indicators:* In order to asses the quality of the path planning module, its generated paths are examined with regard to several quantitative indicators. The most simple indicator is the path length $s_p$. Indicators that characterize the complexity of the planned path are the number $n_w$ of waypoints $w$, the variance $\mathrm{var}(\angle(w^1,\phi_r))$ of the angular deviation

$$\angle(w^1,\phi_r) = \left|\arctan(w^1_y, w^1_x) - \phi_r\right| \quad \in [0,\pi] \tag{6}$$

between next waypoint $w^1$ and robot orientation $\phi_r$, and the cumulative sum of the angular deviation

$$\mathrm{cad} = \sum_{i=1}^{n_w}\angle(w^i, w^{i-1}) \tag{7}$$

between consecutive $w$ in the path, where $\arctan(w^0) = \phi_r$. Finally, the number of waypoints $n_v$ which satisfy a maximum clearance constraint is considered. This can be acquired by using for example distance transformation algorithms.

These metrics can be applied to any global planner which generates paths consisting of a sequence of waypoints. The planning module of the *ACE* robot [15] performs an A* search on a hybrid graph composed of nodes extracted from a bounding box structure and a Voronoi graph. Since Voronoi graphs belong to the family of distance transformation algorithms, the corresponding waypoints satisfy the maximum clearance constraint.

*3) Execution Indicators:* The execution efficiency of a performed navigation task can be evaluated by observing the execution time and the consistency of the path. More specifically, the robot speed $v_r$ and the variance of the robot orientation $\mathrm{var}(\phi_r)$.

All aforementioned indicators have been chosen to represent the internal system state. In the following it is described, how mutual influence among indicators can be learned and used to infer from the internal state about the external world.

## V. LEARNING BAYESIAN NETWORK STRUCTURE FROM DATA

In order to find out whether and to what extent the above performance indicators interact with each other, a Bayesian Network (BN) is learned from experimental data. The topology of the network is unknown beforehand, but the system is fully observable by the data. In order to find the network structure that models the data best, a search through the space of possible structures is performed using a likelihood heuristic.

BNs are well-established tools for representing uncertain relations between several random variables [18]. A BN is an annotated directed acyclic graph, that encodes a joint probability distribution over the set $X = \{X_1,\ldots,X_n\}$ of the random variables described above. Formally it is a tuple $B = \langle G,\Theta\rangle$, where $G$ is a Directed Acyclic Graph (DAG) whose vertices correspond to the random variables. A DAG implies conditional independence of each variable $X_i$ and its non-descendants, given its set of parents $\mathrm{Pa}(X_i)$. $\Theta$ represents the set of parameters that define the transition between nodes. It contains a value $\theta_{i,j,k} = p(X_i = k_i|\mathrm{Pa}(X_i) = j_i)$ for each possible value $k_i$ of $X_i$ and each possible set of values $j_i$ of $\mathrm{Pa}(X_i)$. The conditional probability distribution of each node is represented in a Conditional Probability Table (CPT).

Since there is no a priori transition information, the space of possible DAGs is super-exponential in $n$, the number of variables described, and is given by

$$G(n) = \sum_{k=1}^{n}(-1)^{k+1}\binom{n}{k}2^{k(n-k)}G(n-k). \tag{8}$$

For the ten indicators discussed in the previous section the search space contains $4.2\cdot10^{18}$ graphs and cannot be exhaustively searched. Therefore a Markov Chain Monte Carlo (MCMC) [19] search is performed. The scoring function used for the search is the Bayesian Information Criterion (BIC) [20], which is a function of the log likelihood of the structure according to the training data penalized by the complexity of the structure. The number of samples is chosen so that the search converges to an ordering of nodes close to the global optimum. To further improve the results of the MCMC search the well-established local greedy search algorithm K2 [21] is used. The local search is initialized with the node ordering acquired by the MCMC search.

The K2 search algorithm starts from an empty set of nodes. Parents are added incrementally and the one whose addition increases the score of the resulting structure most, is kept. The algorithm stops adding parents to the node, when it is no longer possible to increase the BIC score of the structure.

In the next section, information-theoretic criteria are used to evaluate the coherence between the indicators within the learned network.

## VI. INFORMATION THEORETIC-CRITERIA

The learned structure of the net provides a first qualitative view on the mutual interactions among the indicators. In the following, information-theoretic criteria [22] are applied in order to derive also a quantitative measure. Once the structure of the net is learned, the CPTs can be computed by using the experimental data. For each pair of indicators $X_i, X_j$, the mutual information

$$I(X_i,X_j) = \sum_j p(j)\sum_{k_i} p(k_i|j)\log\frac{p(k_i|j)}{p(k_i)} \tag{9}$$

is derived. Intuitively, mutual information measures the information that $X_i$ and $X_j$ share, i.e. to what extent knowledge about the one of these variables reduces the uncertainty about the other. For instance, if two variables are independent then knowledge about one of them does not give any information about the other. Consequently their mutual information is zero.

However, in order to make comparisons between different pairs of variables a distance metric is required. In this respect the conjunctional entropy

$$H(X_i,X_j) = H(X_i|X_j) + H(X_j) \quad (10)$$

is additionally calculated, where $H(X) = -\sum_{k\in X} p(x)\log p(x)$ is the entropy of the random variable $X$. The conjuctional entropy measures the uncertainty about the two variables. The final distance metric is then derived by

$$0 \leq \eta(X_i,X_j) = \frac{I(X_i,X_j)}{H(X_i,X_j)} \leq 1. \quad (11)$$

It can be proven [22] that $\eta$ satisfies all properties of a metric such as the triangle inequality, non-negativity and symmetry. If two variables are independent then $\eta(X_i,X_j) = 0$, whereas when the variables are fully dependent and knowledge about the one completely reduces the uncertainty about the other $\eta(X_i,X_j) = 1$.

In the next section the outcome of a field experiment and the application of the described methods is presented.

## VII. EXPERIMENTAL RESULTS

In order to validate the proposed method, system interdependence analysis has been performed by using experimental data which was gathered during autonomous navigation in an urban area by the Autonomous City Explorer (*ACE*) robot. The objective of the experiment was for the *ACE* robot to reach Marienplatz, the central square of Munich, starting from the Technical University of Munich. This is a distance of approximately 1.5 km in the most active part of downtown Munich. The robot did not have prior map knowledge or GPS and relied only on interactions with passers-by to get directions and on its on-board sensors in order to navigate safely. The experiment was conducted successfully on 31 August 2008. The route can be seen in Fig. 2.

The *ACE* robot is based on a differential drive platform. A laser range finder is used for navigation. A SLAM module running at 2 Hz, was used to build a large occupancy grid map on-line. Parts of 200 m×200 m of the acquired map are illustrated in Fig. 2 (a2)-(b2) with a resolution of 15 cm. A part of the occupancy grid around the robot, is used for path planning. Replanning was performed at 2 Hz. More details on the experimental platform as well as the SLAM and planning algorithms, can be found in [15].

In order to quantitatively evaluate the performance and the influence of environmental variations to the system, the presented method has been applied to two exemplary situations encountered during the outdoor field experiment. The first situation, illustrated in Fig. 2(a), demonstrates navigation on a sidewalk in a less populated district. The second situation is illustrated in Fig. 2(b) and is a typical example of navigation in a densely populated pedestrian zone.

Several considerable differences exist between the two settings. In the first, which is referred to as *Sidewalk*, the moving ability of the robot is constrained by the narrow sidewalk but the dynamic characteristics of the environment are low. In the second case, referred to as *Pedestrian zone*, the environment is extensive but primarily characterized from high dynamics and local complexity. This is already observed from the indicator values. Part of them is shown in Fig. 3. For example in the *Pedestrian zone* the map uncertainty $H^m$ and

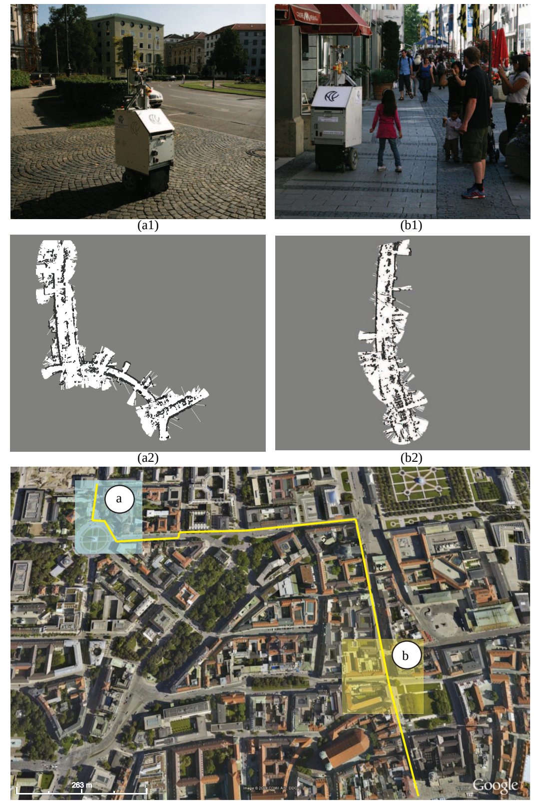


Fig. 2. Bottom: Downtown area of Munich. The route of the robot from the Technical University of Munich to Marienplatz is indicated by the yellow line. The blue box indicates the (a) *Sidewalk* situation and the yellow box the (b) *Pedestrian zone*. Top: (1) Corresponding pictures and (2) the respective maps generated by the *SLAM* module during navigation.

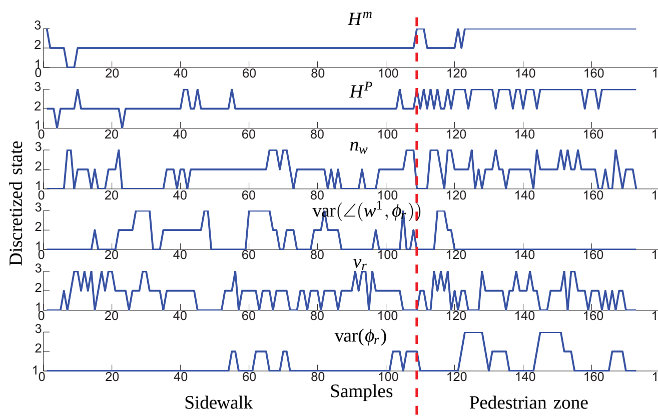


Fig. 3. Discretized indicator (vertical axis) values extracted from experimental data, for two different environments. The dashed line indicates the transition between the environments. The horizontal axis shows the consecutive sample number.

robot orientation variance $\mathrm{var}(\phi_r)$ have mean values that are 43% and 45% higher, respectively. The same applies to their variance which is 6.3 and 6.5 times higher in the *Pedestrian zone*. Intuitively this can be explained by the lower dynamics in the *Sidewalk*. Before the structure of the BN is learned, the data must be discretized and transformed into a predefined number of states. For the following results a discretization of three steps was used for all indicators.

As described in Sec. V, a MCMC search – using 2000 samples to converge – was performed on the preprocessed data to acquire the node order for the BN. The overall

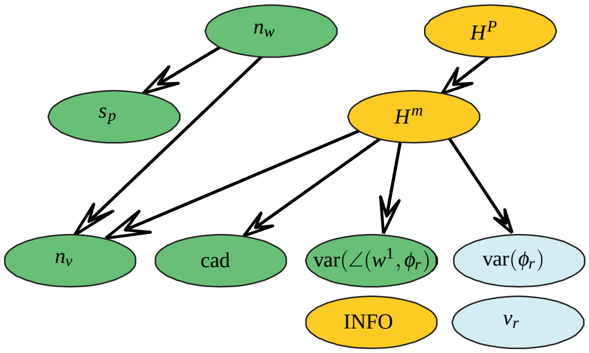


Fig. 4. Directed Acyclic Graph (DAG) learned with MCMC and K2, showing the relations between the perceptual (yellow), planning (green) and execution (blue) indicators.

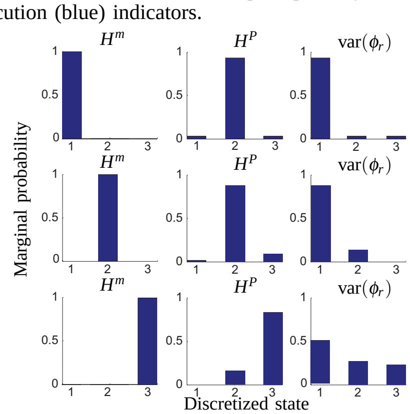


Fig. 6. The marginal distributions of the dependent indicators $H^P$ and var($\phi_r$) as calculated from the learned BN, for assigned values of $H^m$.

best BIC scoring was -1415. Using this ordering, the K2 algorithm generated the final BN, shown in Fig. 4, which achieves a higher BIC score of -1329. This corresponds to an improvement of approximately 6%. The resulting BN indicates the interdependencies between the indicators but cannot express the intensity of these relations. For that reason information-theoretic criteria are applied, as described in Sec. VI.

The learned structure was utilized to train a BN with all the data. Sequential Bayesian parameter updating was performed and the respective CPTs were acquired for the network. An implementation based on the Bayes Net Toolbox for Matlab [23] was used. The distance metric given by (11) is calculated for each possible pair of indicators. The results are illustrated in Fig. 5 by the green solid line.

A strong interdependence of $H^m$ on $H^P$, cad, var($\angle(w^1,\phi_r)$) and var($\phi_r$) is observed. The relation between $H^m$ and $H^P$ is obvious, since without map knowledge it is impossible for the robot to localize itself. Also the influence of $H^m$ on the planning indicators is intuitive, since the path quality is directly dependent on the used map. Map knowledge influences the planned path and therefore the motion of the robot, as reflected by the dependency between $H^m$ and var($\phi_r$). Furthermore $n_w$ is strongly interconnected to $n_v$ and $s_p$, which can be ascribed to the fact that both of them are indicators for the complexity of the calculated path.

The influence of $H^m$ to other indicators can be further quantified by examining the marginal distributions of the affected indicators by setting $H^m$ to specific states. Fig. 6 illustrates the marginal distributions which are calculated from the learned BN by applying Bayesian inference, for all assigned values of $H^m$. When map uncertainty increases, $H^P$ also increases. The motion of the robot becomes more variable as indicated by the uniformly distributed predicted states of var($\phi_r$).

The indicators INFO and $v_r$ show no influence from and to other indicators. This depicts that these indicators cannot give any information about the internal system state or the influence of the environment on the system. The complexity of the system and the application domain cannot be captured by simple and purely local indicators.

In order to asses the environmental influence on the indicators, two additional BNs are trained using the data from the *Sidewalk* and the *Pedestrian Zone* respectively. A comparison of $\eta$, which is shown in Fig. 5, reveals the differences for the two scenes. A stronger influence of $H^m$ on var($\angle(w^1,\phi_r)$) and var($\phi_r$) in the *Pedestrian zone* is identified. The presence of moving people results in higher map uncertainty, less directed, i.e. more variable planned path and consequently more complex robot motion. On the other hand, $n_v$ is stronger related to $n_w$ in the *Sidewalk* scene. In this situation the robot has to navigate through narrow passages, where a maximum clearance path is desired. Consequently, the nodes of the Voronoi graph are more often used.

In summary, the interdependence analysis of system state indicators and the environment, identified map uncertainty $H^m$ as an indicator with very strong influence on the system. Consequently, the intuitive assumption is verified that knowledge of the environment – in this case map knowledge – is a crucial factor for the robustness of an autonomous robotic system. Also it is shown that simpler and local complexity indicators such as $v_r$ and INFO cannot characterize the behavior of the *ACE* robot. In general, by using the proposed method for system analysis, several indicators can be tested in respect to their representation ability. By using the learned BN and inference techniques, predictions can be made about the behavior of performance indicators given the values of others as evidence. Furthermore, the method enables also the reverse interpretation. By observing the internal robot state conclusions about the current environmental situation can be drawn. This way, the robot would be able to asses the situation and the effect that changes in performance indicators of specific system components have on the rest of the system. Such information could be traced back into the on-line behavior selection, used for any kind of on-line learning techniques or at least considered during the off-line system design.

## VIII. CONCLUSIONS AND FUTURE WORKS

A method for quantitative system interdependence analysis has been introduced. The coherence between several performance indicators of different system components, as well as the influence of environmental parameters on the system, can be learned and quantitatively evaluated. This way the robot can anticipate failures, by predicting the

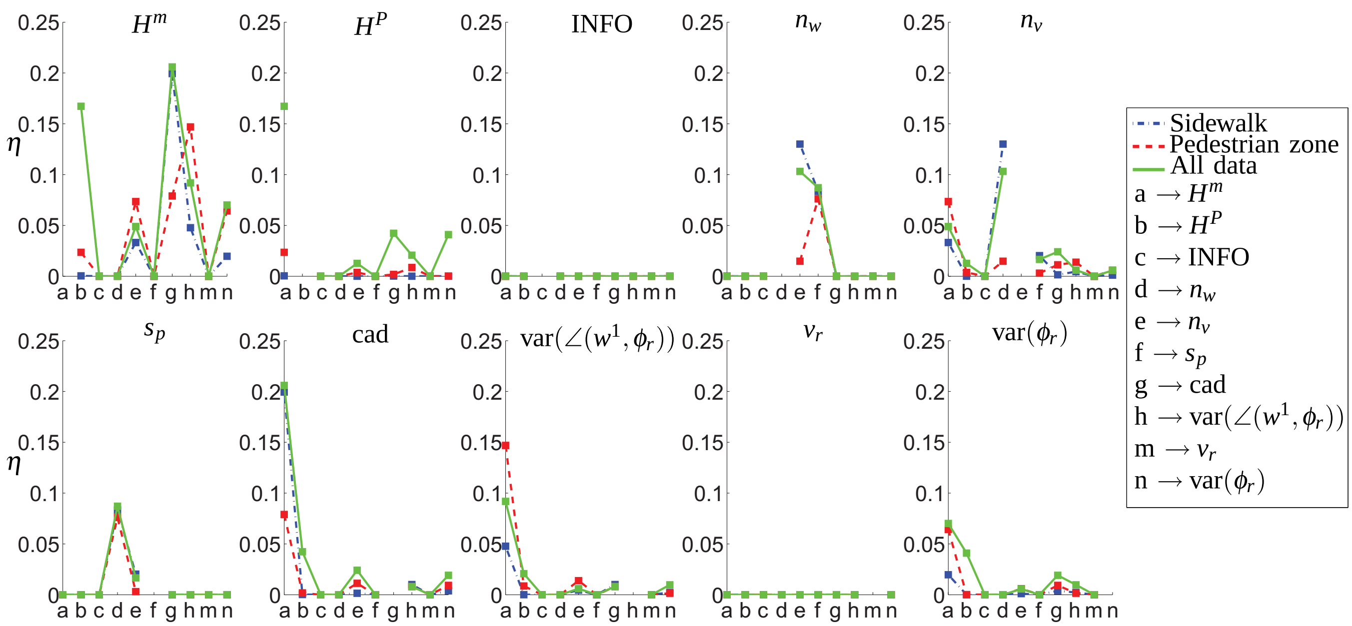


Fig. 5. Learned dependency values $\eta(X_i, X_j)$ (vertical axis) for all indicators, where the i'th graph shows the dependencies of indicator i to all indicators j (horizontal axis). Since $\eta(X_i, X_i) = 1$, these values were skipped for illustrative purposes.

effects that its actions would have and correctly adjust its behavior. Performance evaluation of an autonomous robot navigating in two different urban environments has been conducted and results demonstrated the applicability of the proposed approach. The presented approach is a possibility to identify the limitations of an autonomous robotic system. The complexity of the environment determines the requirements to the robotic hardware and algorithms in order to perform a given task. Conversely, the capabilities of a robotic system define the environments where it can operate and the tasks it can handle.

Further extensions comprise the evaluation of more indicators. Incorporating the outcome of the proposed interdependence analysis to the on-line system operation, will enable the system to reason about the current situation and adjust its behavior accordingly.

## IX. ACKNOWLEDGMENTS

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