

Memoryless Input-Output Encoding for Networked Systems with Unknown Constant Time Delay

Tilemachos Matiakis and Sandra Hirche

Abstract—Communication time delay in a networked control system (NCS) degrades the performance and may lead to instability. The time delay value depends on the network configuration, e.g. number of nodes and is not exactly known during the controller design stage. Based on a novel delay-independent input-output approach the problem of unknown constant time delay is addressed in this paper. The plant and the controller input and output are encoded by a memoryless transformation into new variables which are transmitted over the communication channel. Thereby finite gain L_2 stability is achieved. The performance of the memoryless encoding approach, validated in simulations and experiments, is superior over two other approaches, the standard small gain and a lead-lag control approach.

I. INTRODUCTION

Due to the increasing complexity of control systems and novel requirements, such as decentralized control, the traditional control architecture with point to point interconnection between the plant and the controller is more and more replaced by networked control systems (NCS). In NCS the components are spatially separated and the control loop is physically closed through a communication network. Significant advantages are reduced wiring and cost, increased modularity, easier maintenance and system diagnosis as well as higher flexibility and reconfigurability, meaning that nodes can be added or removed without additional wiring effort. However, the signal transmission between the plant and the controller in NCS can no longer be regarded ideal. Time delay, data drop out and limited communication resources in the closed loop system represent challenges for the control synthesis, see e.g. [1] for an overview. The transmission characteristics depend on the number of active nodes sharing the communication channel and are typically not exactly known during the controller design stage.

In this work the problem of unknown constant time delay is addressed. It is well-known that time delay in a closed control loop degrades the performance and can lead to instability. Most common in the time delay systems literature are state-space approaches based on a Lyapunov-like stability analysis, see [2], [3] for an overview. In practice, however, the input-output model of a plant is often easier to

obtain than the full state space model. Delay-independent input-output approaches are rarely applied in practice as the necessary small gain condition - the open loop gain less than one - renders these approaches very conservative. For instance, systems with integrators in the open loop cannot be considered, resulting thus in a poor tracking performance to a reference input. The input-output approaches considered in [4], [5] assume the time delay to be known.

As an alternative, in this paper a novel delay-independent input-output approach is investigated. In contrast to standard approaches plant and controller output are not directly sent over the network. Here, the transmitted variables are obtained by encoding plant and controller input and output via a memoryless time-invariant linear transformation, recently introduced in [6]. The approach is applicable to the class of input-feedforward-output-feedback-passive (IF-OF) plants comprising nonlinear, time-varying, and some “well-behaving” unstable input-output relations. As a result of the input-output transformation the IF-OF property of the plant is preserved through the network independently of the time delay. Any controller-plant pair satisfying a certain finite gain L_2 stability condition *without time delay*, is finite gain L_2 stable also *with arbitrarily large constant time delay*. The knowledge of the time delay value is not needed for the synthesis of a stable system. Furthermore, the system can be designed such that the transient performance is rather insensitive to the time delay value [6]. This paper addresses performance and design issues. A comparison with two other approaches, the standard delay-independent small gain and a delay-dependent lead-lag control approach, is conducted showing the superior performance of the input-output encoding approach. Experiments with a one degree-of-freedom robotic system convincingly validate the proposed approach.

The remainder of this paper is organized as follows: In Section II. some background is presented, followed by the system description with the input-output transformation in Section III. Performance and design issues are discussed in Section IV. followed by the performance comparison in simulation and experiment in Section V. and conclusions in Section VI.

II. BACKGROUND

A. L_2 Stability

Let $\|u\|_{L_2}$ denote the L_2 norm of a piecewise square-integrable function $u(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}^m$ with \mathbb{R}_+ being the set of non-negative real numbers and \mathbb{R}^m the Euclidean space of dimension m . The truncation of $u(\cdot)$ up to the time t is denoted by $u_t(\cdot)$, and the extended space of Lebesgue

T. Matiakis is with the Institute of Automatic Control Engineering, Technische Universität München, D-80290 Munich, Germany t.matiakis@tum.de

S. Hirche is a visiting researcher with Fujita Lab., Dept. of Mechanical and Control Engineering, Tokyo Institute of Technology, 152-8552 Tokyo, Japan s.hirche@ieee.org

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to the controller output $y_c \in L_{2e}$, see Fig. 1. The control error is defined as $e = w - u_c$ with $w \in L_{2e}$ the desired value and u_c the lefthand side output of the communication channel. The plant is connected to the controller through a communication network. However, instead of directly transmitting plant (controller) output over the communication channel, a linear combination of plant (controller) output and input is sent to the corresponding receiver side. Similarly to [6] a memory-less transformation is favored due to the limited computational power on the plant side. The transformation matrix $M \in \mathbb{R}^{2 \times 2}$ acts on the plant and the controller input-output vector $z_p^T = [u_p y_p]$ and $z_c^T = [y_c u_c]$, respectively

$$s_r = Mz_p \quad \text{and} \quad s_l = Mz_c, \quad (4)$$

where $s_r^T = [u_r v_r]$ and $s_l^T = [u_l v_l]$ represent the values transmitted over the communication channel, see Fig. 1. The transformation matrix M is parameterized as a rotation matrix R and a scaling matrix B

$$M = RB \quad \text{with} \\ R = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}; \quad B = \begin{bmatrix} \sqrt{b} & 0 \\ 0 & \frac{1}{\sqrt{b}} \end{bmatrix}, \quad (5)$$

with the rotation angle $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and the scaling parameter $b > 0$, both constant. The mapping by M is a bijection; it belongs to the class of *special linear transformations*, i.e. $\det M = 1$, hence is non-singular, an inverse exists. Note that for $M = I_2$, with I the identity matrix, the standard approach without transformation is recovered.

The network is modelled with a forward time delay T_1 (plant to controller channel) and backward time delay T_2 . The value u_l is transmitted from the controller to the plant side arriving as value $u_r(t) = u_l(t - T_1)$. Accordingly for the backward channel $v_l(t) = v_r(t - T_2)$ holds. It is assumed that $u_r(t) = 0 \quad \forall t \in [0, T_1]$ and $v_l(t) = 0 \quad \forall t \in [0, T_2]$. For further reference, the following subsystems: $v_r = h_1(u_r)$ and $u_c = h_2(y_c)$ are defined, see also Fig. 1.

Throughout the paper we assume that the closed loop system is well posed, i.e. for each input signal $w \in L_{2e}$ there exists a unique solution for the signals $e, u_c, y_c, u_l, v_l, u_r, v_r, u_p, y_p$ that causally depends on w . Further, we assume the following system properties:

- 1) The time delays T_1, T_2 are arbitrarily large but constant.
- 2) Plant h_p and controller h_c are IF-OPF with δ_i, ε_i with $i \in \{p, c\}$ satisfying Proposition 1, i.e. the feedback interconnection *without time delay* is finite gain L_2 stable.

B. Stability

In the following the most important stability results are briefly reviewed, for a detailed analysis, proofs and further discussion refer to [6]. For the following consideration the closed loop system is decomposed into the negative feedback interconnection of subsystem h_2 , and the controller h_c , see Fig. 1. By proper selection of the transformation angle θ the subsystem h_2 is IF-OPF with exactly the plant IF-OPF parameters δ_p, ε_p independently of time delay.

Theorem 1: [6] The subsystem h_2 is IF-OPF with δ_p, ε_p if and only if the following holds

$$\cot 2\theta = \varepsilon_p b - \frac{\delta_p}{b}, \quad (6)$$

and

$$\sin(\theta)\cos(\theta) - \frac{\delta_p}{b}\cos^2(\theta) - \varepsilon_p b\sin^2(\theta) \geq 0. \quad (7)$$

The second condition (7) merely defines one of the two existing solutions of θ in (6). As this solution exists for each $b > 0$, this scaling parameter can be chosen freely to meet performance requirements.

From this result it is straightforward to conclude finite gain L_2 stability of the closed loop system.

Corollary 1: [6] The closed loop system with the transformation (5) and the transformation parameters satisfying Theorem 1 is finite gain L_2 stable independently of the time delay.

Crucial point of the proposed approach is that the input-output transformation transforms the IF-OPF plant h_p into a finite gain L_2 stable subsystem h_1 such that

$$\|v_{r,t}\| = \|h_1(u_{r,t})\| \leq \gamma_{h_1} \|u_{r,t}\| \quad \forall t; \quad \gamma_{h_1} \geq 0, \quad (8)$$

holds. The derivation of the gain γ_{h_1} can be found in [6]. A constant time delay operator does not alter this L_2 gain γ_{h_1} during transmission, since it has a L_2 gain equal to one. The inverse transformation M^{-1} , i.e. the decoding, is a bijection as well, and therefore the exact IF-OPF property δ_p, ε_p of the plant is recovered at the subsystem h_2 . As an important result, the feedback interconnection of any controller-plant pair satisfying the finite gain L_2 condition from Proposition 1 *without time delay* is finite gain L_2 stable for *arbitrary large time delay* by using the proposed encoding approach.

An intuitive geometrical interpretation can be given for a LTI system. The input-output transformation represents a LFT, see Section II-C, from the plant transfer function $G_p(j\omega)$ to the transfer function $G_1(j\omega)$ of the subsystem h_1

$$G_1(j\omega) = \frac{\cos(\theta)G_p(j\omega) - b\sin(\theta)}{\sin(\theta)G_p(j\omega) + b\cos(\theta)} \quad (9)$$

as straightforward derivable from the transformation equations (4). This LFT maps the circle that the Nyquist plot of the IF-OPF plant $G_p(j\omega)$ lies in, to a finite L_2 gain circle with center at the origin, see Fig. 2. The arbitrary large time delay in series connection with G_1 does not alter this circle as it only rotates each point around the origin as observable from the transfer function $G_1(j\omega)e^{-j\omega T}$ where $T = T_1 + T_2$. The inverse transformation, since it is a bijection, maps the finite L_2 gain circle back to the initial IF-OPF circle of the plant. The IF-OPF property of the plant is restored to the subsystem h_2 independently of the time delay.

Remark 1: In case of an unstable plant, the plant is locally pre-stabilized with the right hand input-output transformation. This is clear from the fact that each IF-OPF plant results in a finite gain L_2 stable system after the right hand transformation as observable from (8).

Remark 2: In case of a passive plant the LFT (9) with $\theta = 45^\circ$ recovers the well-known scattering transforma-

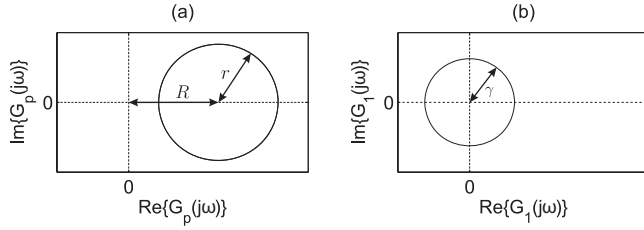


Fig. 2. (a) Nyquist plot area of the IF-OFP plant G_p . (b) Nyquist plot area of the finite gain L_2 stable subsystem $G_1(s)$.

tion widely used for the stabilization of teleoperation systems in the presence of arbitrarily large constant time delay [10], [11]. The scattering transformation maps the right half plane to the unit circle, i.e. a passive system to a finite gain L_2 stable system with L_2 gain one.

Remark 3: The input-output encoding makes the closed loop systems stability unaffected by any causal operator with a L_2 gain $\gamma \leq 1$ inserted within the loop of encoded variables. In fact, many scattering based approaches to cope with communication effects such as time-varying delay [12] and packet loss [13], [14] are based on the small gain principle within the scattering variable domain. These approaches are straightforward to apply here.

IV. PERFORMANCE AND DESIGN ISSUES

In the following some performance issues; the closed loop system is assumed to be finite gain L_2 stable.

A. Steady State Behavior and Zero Delay Case

The steady state behavior with input-output encoding and time delay is equivalent to the one without input-output encoding and without time delay, as in the steady state $s_l = s_r$ holds, hence $z_c = M^{-1}s_l = M^{-1}s_r = M^{-1}Mz_p = z_p$. The same, $z_c = z_p$, holds in the zero delay case, meaning that the system reduces to that without time delay and without input-output encoding; nominal performance is achieved.

In terms of steady state error the proposed approach clearly outperforms the standard small gain approach. The standard small gain approach, e.g. for LTI systems, requires $|G_c(j\omega)G_p(j\omega)| < 1, \forall \omega$, i.e. free integrators in the open loop are not allowed. This leads to a rather large steady state error, e.g. $|e(t)|_{t \rightarrow \infty} > \frac{1}{2}|w|$ for a reference step input w . In the proposed approach free integrators in plant or controller do not necessarily contradict Corollary 1. This can be easily seen using counterexamples, e.g. $G_p(s) = \frac{s+1}{s+2}$ and the stabilizing controller $G_c(s) = 1 + \frac{1}{s}$ where the values of δ, ε are easily defined from the Nyquist plot. Thus, steady state error zero and independent-of-delay stability can be guaranteed simultaneously. Further, in [6] it has been shown that the transformation and the controller can be designed such that the closed loop performance is rather insensitive to different time delay values.

B. Design Procedure

Based on the stability result from the previous section a controller design procedure is proposed.

- 1) Define the plant δ_p, ε_p values, (e.g. Nyquist plot).

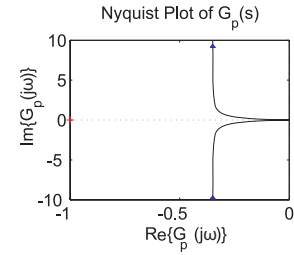


Fig. 3. Nyquist plot of the plant transfer function $G_p(j\omega)$.

- 2) From Theorem 1 define the minimum values for δ_c, ε_c of the controller so as to guarantee stability for the closed loop system.
- 3) Find a controller and a value for the scaling parameter b of the transformation, so as to satisfy performance criteria under the constraints of δ_c, ε_c and the angle θ of the transformation given according to Theorem 1.

In step 1) it is desirable to determine the tightest bound for δ_p, ε_p in order to avoid conservatism. However, this is non-trivial in general. How to define the optimal controller with respect to some performance criteria in step 3) is still an open problem and subject of current research.

V. PERFORMANCE COMPARISON

In order to show the efficacy of the input-output encoding approach a comparison is performed with a controller satisfying the classical small gain approach for delay-independent stability, and with the delay-dependent lead-lag controller. It should be mentioned that the latter comparison is unfair since delay-dependent methods usually lead to less conservative results. In lack of a systematic controller design methodology with input-output encoding a general performance comparison is difficult at this stage. Instead, a case study is conducted here with the controller designed through numerical optimization. A one degree-of-freedom robotic system later used for experimental validation is considered with the approximated transfer function

$$G_p(s) = \frac{36}{s^2 + 10.15s}. \quad (10)$$

A. Controller Design

For fair comparison the controller for all approaches are designed by numerical optimization using the integrated squared tracking error

$$J = \int_0^{t_f} e^2(\tau) d\tau,$$

as cost function. The numerical optimization is performed for a nominal time delay of $T = 300\text{ms}$ over a horizon of $t_f = 5\text{s}$ using `fmincon` of the Matlab optimization toolbox. As basic structure for the controller a lead-lag element

$$G_c(s) = k \frac{s+a}{s+c},$$

is considered where $k, a, c > 0$ are parameters to be determined by numerical optimization. The exact design procedure for all three cases is explained in the next.

1) *Input-Output Encoding*: Following the design steps given in Section IV-B, first the values δ_p, ε_p are determined from the Nyquist plot of (10) presented in Fig. 3. Clearly, $\text{Re}\{G_p(j\omega)\} > -0.3494$, further $\text{Im}\{G_p(j\omega)\} \rightarrow \infty$ as $\omega \rightarrow 0$ giving $\delta_p = -0.3494$, $\varepsilon_p = 0$, see Section II-C. For stability based on Theorem 1 $\delta_c > 0$ and $\varepsilon_c > 0.35$ must be satisfied for the controller. The Nyquist plot of the lead-lag controller is a circle with center on the real axis at R and radius r given by

$$R = \frac{k(a+c)}{2c}; \quad r = \frac{k|a-c|}{2c}. \quad (11)$$

From these values, using the equations from Section II-C, follows $\delta_c = \frac{R^2 - r^2}{2R} > 0$, which is always satisfied as long as $k, a, c > 0$. Furthermore, (11) and the requirement $\varepsilon_c > 0.35$ leads to

$$\frac{k(a+c)}{2c} < 1.4286, \quad (12)$$

imposing a stability constraint to the lead-lag controller parameter values. The resulting constrained optimization problem can be formulated as $\min_{k,a,c,b,\theta} J$ subject to the equality constraint (6), and the inequality constraints (7), (12), and $k, a, c, b > 0$. The numerical optimization gives the controller for the input-output transformation

$$G_{tr}(s) = \frac{1.6160(s + 2.5833)}{s + 3.3628},$$

with all constraints satisfied and $b = 0.1816$, $\theta = 13.73^\circ$. The optimal cost function value is $J = 0.2927$.

2) *Classical Small Gain Controller*: In order to achieve delay-independent stability without input-output encoding the small gain condition has to be satisfied requiring $|G_0(j\omega)|_\infty = |G_p(j\omega)G_{sg}(j\omega)|_\infty < 1 \forall \omega$ with G_{sg} the small gain controller transfer function and $|G_0(j\omega)|_\infty$ the H_∞ norm of the open loop transfer function G_0 . As the small gain condition cannot be satisfied with the integrator in the plant (10) a high pass component is added to the lead-lag controller. Further, the controller gain k is fixed to $k = 1/|G_0(j\omega)|_\infty$. The optimization problem is given by $\min_{a,c} J$ subject to the inequality constraint $a, c > 0$. The numerical optimization gives the controller

$$G_{sg} = \frac{0.1855(s + 5.3574)}{(s + 3.5258)} \frac{s}{0.001s + 1},$$

where the latter part represents the high pass component. The optimal cost function value is $J = 1.1192$.

3) *Delay-Dependent Lead-Lag Controller*: Similarly to the previous case the optimization problem is represented by $\min_{k,a,c} J$ subject to the constraint $k, a, c > 0$ resulting in the controller

$$G_{ll} = \frac{2.6136(s + 2.9028)}{(s + 14.3436)},$$

with the optimal cost function value $J = 0.2917$.

B. Simulations

The performance is not only compared for the nominal time delay, but for different values as well, in order to

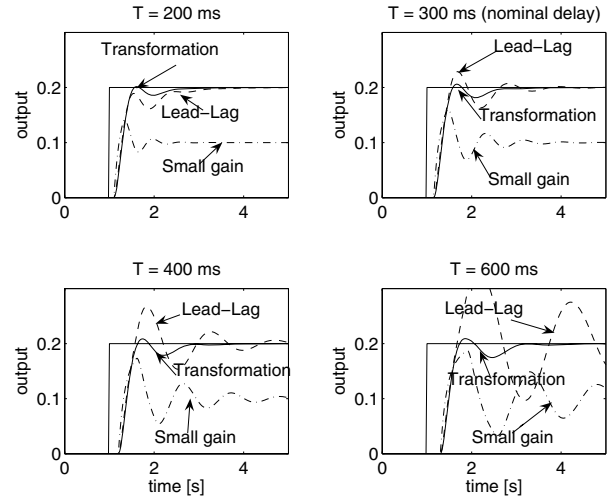


Fig. 4. Simulation results: Step response with input-output encoding (transformation), small gain based controller and lead-lag controller for various time delay values

TABLE I
SIMULATION RESULTS: $\pm 5\%$ SETTLING TIME

Time Delay (ms)	200	300	400	600
I/O Encoding	1.18s	1.36s	1.53s	1.85s
lead-lag	1.32s	1.53s	3.11s	-
Small gain	1.05s	1.87s	3.38s	-

validate the low sensitivity of the closed loop performance with input-output encoding to different time delay values as indicated in [6]. A step input of magnitude 0.2 rad and round trip time delay values $T \in \{200, 300, 400, 600\}$ ms are considered.

From the simulation results in Fig. 4 we observe that the input-output (I/O) encoding approach outperforms the other approaches, not only for the nominal time delay, but also for the other time delay values. The overshoot is small and only slightly affected by the time delay value; the settling time is much smaller than with the other two approaches and does not change significantly for different time delay as well, see also Table I. The small gain controller gives a steady state error of approximately 50% in all cases. With increasing time delay the settling time grows significantly. The lead-lag controller without input-output encoding performs well at the nominal time delay but the performance is, as expected, sensitive to the time delay value: Still stable for the maximum time delay value tested it shows significant overshoot and settling time when the time delay increases.

C. Experiments

The experimental testbed consists of the 1DOF pendulum shown in Fig. 5, connected to a PC running under RT Linux. The original design of the pendulum can be found in [15]. The DC-motor current, resulting in a torque, is provided by the PWM amplifier operated under current control. The reference signal is given by a voltage from the D/A converter output of the I/O board. The position of the lever is measured by an optic pulse incremental encoder and processed by

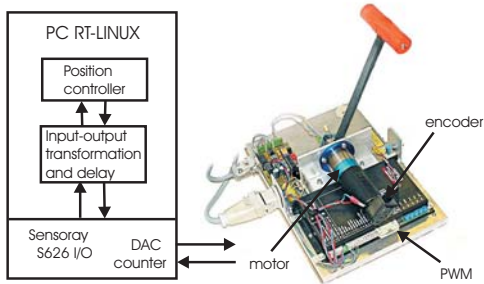


Fig. 5. Experimental testbed with one degree-of-freedom robotic system.

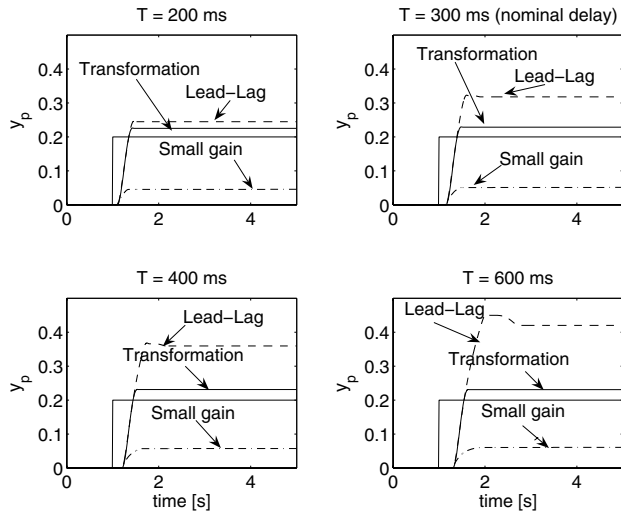


Fig. 6. Experimental results: Step response with input-output encoding (transformation), small gain based controller and lead-lag controller for various time delay values.

a quadrature encoder on the I/O board. The control loop including the controller, the input-output transformation, and the time delays are implemented in MATLAB/SIMULINK blocksets. Standalone realtime code is generated directly from the SIMULINK model. The experiments are performed with a sampling time interval $T_A = 1$ ms.

The experimental results are presented in Fig. 6. The proposed approach outperforms even the delay-dependent lead-lag controller for nominal time delay. Not only the smallest steady state error appears, see also Table II, the performance is furthermore rather insensitive to the time delay value. The small gain controller has, as in the simulations, a very high steady state error. The steady state error with the delay-dependent lead-lag controller is high as well and increases with increasing time delay. The observed steady state error in all cases is most probably due to unmodelled nonlinearities such as friction and backlash. In summary, the proposed input-output encoding approach is convincingly validated.

TABLE II

EXPERIMENTAL RESULTS: STEADY STATE ERROR IN %

Time Delay (ms)	200	300	400	600
Transformation	12.6	14.4	15.5	15.4
lead-lag	20.8	46	81.9	109.9
Small gain	77.1	74.3	71.3	69.6

VI. CONCLUSIONS

A novel input-output approach for NCS guaranteeing finite gain L_2 stability independently of the constant time delay is investigated in this paper. Input and output of plant and controller are encoded into new variables which are transmitted over the communication channel. The performance is superior over the classical delay-independent input-output approach with direct transmission of the plant (controller) output employing the small gain theorem. Even a delay-dependent input-output approach is outperformed in a case study based on simulations and experiments. Clearly the input-output encoding approach holds great promise for NCS. Future work is to develop a systematic control design methodology.

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