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A Novel Input-Output Transformation Method to Stabilize Networked Control Systems Independent of Delay

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***Abstract*— In this paper a novel approach for the control of networked control systems (NCS) to achieve input-output stability independent of constant time delay, using the concepts of $\mathscr{L}_2$ stability is proposed. Main feature is a special linear transformation of the input-output space of the controller and the plant. The transformed signals (linear combinations) are sent over the network. Main result of this paper are sufficient stability conditions for general, input-feedforward-output-feedback-passive nonlinear systems. For the linear case with known transfer functions neccessary and sufficient conditions are given.**

***Keywords*— Networked control system (NCS), delay-independent stability, finite gain $\mathscr{L}_2$ stability, input-feedforward-output-feedback passivity**

I. INTRODUCTION

The motivation of the work presented here comes from networked control systems (NCS), where the plant and the controller are spatially separated, and connected through a communication network, see e.g. [1] for an overview. One of the major advantages associated with the use of communication networks for signal transmission in control systems is flexible reconfiguration: Nodes can be added or removed without additional wiring effort. The number of active nodes sharing the communication line has an effect on the communication parameters in terms of communication time delay, packet loss, and available communication bandwidth. In consequence, these communication parameters may not be known exactly or not at all during the controller design stage. In this work the problem of unknown, constant time delay is addressed. Main contribution is a novel method to input-output stabilize NCS with unknown, constant time delay. The plant is represented by an input-output relation, that can be nonlinear and time-varying.

It is well-known that time delay in a closed control loop degrades performance and can lead to instability. In the rich literature on time delay systems mostly state-space approaches are considered, see [2], [3] for an overview. The time delay input-output approaches in [4], [5] assume the time delay to be known. Input-output approaches for linear systems with uncertain time delay are considered in [6], [7]. The classical small gain result requiring the open loop system to have a gain less then one is known to be rather conservative; e.g. systems with integrators in the open loop cannot be considered. In consequence, the tracking performance to a reference input is generally poor.

In this paper an input-output approach is adopted, in lines with [8], [9]. In these seminal works conditions for the open-loop behavior of feedback components are provided that guarantee stability of the feedback interconnection. The main result is stated as follows: “If the open loop can be factored into two suitably proportioned, conic relations then the closed loop is bounded.” The small gain theorem and passivity become special cases.

In our approach we assume plant and controller to be input-feedforward-output-feedback-passive (IF-OFP) relations such that stability for the closed loop system is guaranteed *without time delay*, i.e. the open loop can be factored into suitable conic sectors [8]. The class of IF-OFP systems is very general also including certain “well-behaving” unstable systems. The key idea is to transmit a linear combination of input and output variables (by transformation) instead of direct communication. As a result the IF-OFP property of the plant is preserved through the network independently of the time delay; the closed loop system is stable. Interpretation in terms of conic sectors: By input-output transformation the IF-OFP conic sector is rotated such that a non-conservative finite gain $\mathscr{L}_2$ condition is satisfied for the transformed input-output relations. In fact, we show that the small gain condition is satisfied in the transformed loop, i.e. stability is achieved for arbitrarily large, constant time delay. Interesting properties of the resulting NCS are that the design goals for stability, performance and, insensitivity with respect to time delay uncertainty do not rely on knowledge of the time delay value. Compared to the standard small gain approach the controllers can be tuned rather aggressively similar to the case *without time delay*. The resulting tracking performance is good in a large time delay range. In case of passive subsystems, the proposed approach is equivalent to the well-known scattering transformation [10], [11] widely used for the stabilization of teleoperation systems.

II. PRELIMINARIES

In this paper dynamical systems are considered from an input-output point of view most generally as a causal input-output mapping operator. Notation definitions and some known facts are given in the following.

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A. Notation

Let $\mathscr{L}_{2e}^m$ denote the extended $\mathscr{L}_2$ space of time functions of dimension m with support on $[0,\infty)$. We consider causal mappings $h:\mathscr{U}\rightarrow\mathscr{Y}$, where $\mathscr{U}$ and $\mathscr{Y}$ are appropriate subspaces of $\mathscr{L}_{2e}^m$ and $h(t=0)=0$. The system is supposed to be well defined in the sense that to each element in $\mathscr{U}$ an element in $\mathscr{Y}$ is associated. Let $\|u\|$ denote the $\mathscr{L}_2$ norm of a piecewise square-integrable function $u(\cdot):\mathbb{R}_+\rightarrow\mathbb{R}^m$ with $\mathbb{R}_+$ being the set of non-negative real numbers and $\mathbb{R}^m$ the Euclidean space. The truncation of $u(\cdot)$ up to the time t is denoted by $u_t(\cdot)$. The inner product of the truncated signals u_t, y_t is denoted by $\langle u,y\rangle_t$, hence $\|u_t\|^2=\langle u,u\rangle_t$. For later analysis of linear time-invariant (LTI) systems, we denote by $|G|^\infty$ the H_∞ norm of the transfer function G.

B. Input-Output Stability

Among the variety of input-output stability notions we consider finite gain $\mathscr{L}_2$ stability in this paper.

Definition 1: A dynamical system h is said to be finite gain $\mathscr{L}_2$ stable if there exists a positive constant $\gamma\in\mathbb{R}_+$ such that for each admissible $u\in\mathscr{U}$ and each $t\in[0,\infty)$ we have [12]

$$\|y_t\|\leq\gamma\|u_t\|. \tag{1}$$

The feedback components in this paper are assumed to be input-feedforward-output-feedback-passive (IF-OFP).

Definition 2: A dynamical system h is said to be input-feedforward-output-feedback-passive (IF-OFP) if there exist constants $\delta,\varepsilon\in\mathbb{R}$ such that for each admissible $u\in\mathscr{U}$ and each $t\in[0,\infty)$ we have [12]

$$\langle u,y\rangle_t\geq\delta\|u_t\|^2+\varepsilon\|y_t\|^2. \tag{2}$$

In physical interpretation $\langle u,y\rangle_t$ represents the external energy flow into the system up to the time t. The above input-output description is a generalization of the passivity concept. If $\delta=\varepsilon=0$ then the system is passive, i.e. it does not generate energy. If $\delta=0$ and $\varepsilon>0$ the system is called output feedback strictly passive and if $\delta>0$ and $\varepsilon=0$ input feedforward strictly passive. In both these cases the system dissipates energy. If one or both of the values δ,ε are negative then there is a shortage of passivity in the system. The system can generate energy, but this energy is bounded by the squared $\mathscr{L}_2$ norm of the input and/or the output signal. Note, that IF-OFP is a special case of dissipativity with a special quadratic supply rate [13]–[15].

One important stability result for closed loop systems comes from the IF-OFP property of its subsystems. Consider two IF-OFP systems h_p and h_c satisfying (2) with some δ_i,ε_i with $i\in\{p,c\}$.

Proposition 1: The negative feedback interconnection of h_p and h_c is finite gain $\mathscr{L}_2$ stable if

$$\varepsilon_c+\delta_p>0 \qquad \text{and} \qquad \varepsilon_p+\delta_c>0.$$

Proof: See [12]. ■

Clearly, some of the δ_i,ε_i can be negative if compensated by other positive values. Within the passivity formalism this can be interpreted as balancing passivity shortage and excess passivity between subsystems.

III. PROBLEM FORMULATION

We consider a system consisting of a plant $h_p:\mathscr{U}_p\rightarrow\mathscr{Y}_p$ and a controller $h_c:\mathscr{E}\rightarrow\mathscr{Y}_c$ as mappings from the plant input $u_p\in\mathscr{U}_p\subset\mathscr{L}_{2e}^m$ to the plant output $y_p\in\mathscr{Y}_p\subset\mathscr{L}_{2e}^m$ and from the control error $e\in\mathscr{E}\subset\mathscr{L}_{2e}^m$ to the controller output $y_c\in\mathscr{Y}_c\subset\mathscr{L}_{2e}^m$, see Fig. 1. The control error is defined as $e=w-u_c$ where $w\in\mathscr{W}\subset\mathscr{L}_{2e}^m$ is the reference input and u_c the lefthand side output of the communication channel. The plant is connected to the controller through a communication network. In this paper we propose not directly to transmit plant (controller) output over the communication channel, but a linear combination of plant (controller) output and input. Here a memory-less transformation is favored due to the limited computational power on the plant side. The considered constant transformation matrix $M\in\mathbb{R}^{2m\times2m}$ maps the plant input-output $z_p^T=[u_p^T\,y_p^T]$, $z_p\in\mathscr{U}_p\times\mathscr{Y}_p$, to the righthand side transmitted values $s_r^T=[u_r^T\,v_r^T]$; analogously for the controller side

$$s_r=Mz_p \quad \text{and} \quad s_l=Mz_c \tag{3}$$

with $z_c^T=[y_c^T\,u_c^T]$, $z_c\in\mathscr{Y}_c\times\mathscr{U}_c$ the lefthand side communication input-output, $s_l^T=[u_l^T\,v_l^T]$ the lefthand side transmitted values, see Fig. 1. Note that for $M=I_{2m}$, with I the identity matrix, the standard approach without transformation is recovered.

The network is modelled as a forward time delay operator h_{T_1} (plant to controller channel) and backward time delay operator h_{T_2} (controller to plant channel) with time delays T_1 and T_2, respectively. Accordingly, the relation between inputs and outputs is given by $h_{T_1}:u_r(t)=u_l(t-T_1)$ and $h_{T_2}:v_l(t)=v_r(t-T_2)$. Thus, the value u_l is transmitted over the forward channel and arrives delayed by the time delay T_1 at the plant, now denoted by u_r, analogously for the backward channel. It is assumed that $u_l(t)=0\ \ \forall t\in[-T_1,0]$ and $v_r(t)=0\ \forall t\in[-T_2,0]$. For further reference we define the following 3 subsystems: $v_r=h_1(u_r)$, $u_c=h_2(y_c)$, and $u_l=h_3(v_l)$, see Fig. 1.

Throughout the paper we assume that the closed loop system is well posed, i.e. for each input signal $w\in\mathscr{W}$ there exists a unique solution for the sig-

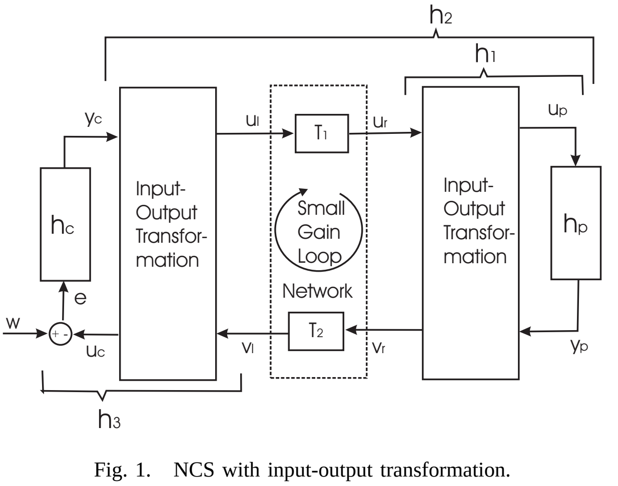

Fig. 1. NCS with input-output transformation.

nals $e, u_c, y_c, u_l, v_l, u_r, v_r, u_p, y_p$ that causally depends on w. Further, we assume the following system properties:

1) The time delays T_1 and T_2 are arbitrarily large but constant.
2) Plant h_p and controller h_c are IF-OFP with δ_i, ε_i with $i \in \{p, c\}$ satisfying Proposition 1, i.e. the feedback interconnection *without time delay* is finite gain $\mathscr{L}_2$ stable.

Clearly, under these assumptions considering the standard approach, i.e. $M = I$, the closed loop system can be unstable. This can easily be verified as shown e.g. in [10] for passive subsystems. The main result of this paper is a delay-independent finite gain $\mathscr{L}_2$ stability condition for the closed loop system of any plant-controller pair satisfying assumption 2) together with the input-output transformation. The result is constructive with respect to the design of the transformation M.

IV. MAIN RESULT

For convenient notation we consider a single-input-single-output plant and controller in the following, i.e. $m = 1$. Where non-ambiguous, the time argument t is dropped for convenience of notation. Before the main result is stated some technical lemmas are given. All proofs are in the Appendix.

Lemma 1: Without loss of generality the domain of δ, ε in IF-OFP systems (2) is considered by $\Omega = \Omega_1 \cup \Omega_2$ with $\Omega_1 = \{\delta, \varepsilon \in \mathbb{R} | \delta\varepsilon < 1/4\}$ and $\Omega_2 = \{\delta, \varepsilon \in \mathbb{R} | \delta\varepsilon = 1/4; \varepsilon > 0\}$.

Accordingly, from now on we consider $(\delta, \varepsilon) \in \Omega$.

Lemma 2: Consider the expressions

$$\begin{aligned} \alpha(\theta_i) &= \sin(\theta_i)\cos(\theta_i) - \frac{\delta_p}{b}\cos^2(\theta_i) - \varepsilon_p b \sin^2(\theta_i) \\ \beta(\theta_i) &= \alpha(\theta_i) + \frac{\delta_p}{b} + \varepsilon_p b \end{aligned} \tag{4}$$

where $\theta_i,\ i \in \{1, 2\}$ are the two solutions of $\cot(2\theta) = \varepsilon_p b - \delta_p / b$ in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $b > 0$, and $(\delta_p, \varepsilon_p) \in \Omega$. Then the following statements are true:
- $\alpha(\theta_1), \beta(\theta_1) > 0$ and $\alpha(\theta_2), \beta(\theta_2) < 0$ if $(\delta_p, \varepsilon_p) \in \Omega_1$,
- $\alpha(\theta_i) = 0,\ \beta(\theta_i) > 0$ if $(\delta_p, \varepsilon_p) \in \Omega_2$.

We are now able to state our main result. The transformation matrix M is parameterized as a rotation matrix R and a scaling matrix B

$$M = RB \quad \text{with}$$
$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}; \quad B = \begin{bmatrix} \sqrt{b} & 0 \\ 0 & \frac{1}{\sqrt{b}} \end{bmatrix}, \tag{5}$$

with the rotation angle $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and the scaling parameter $b > 0$, both constant. The mapping by M is a bijection; it belongs to the class of *special linear transformations*, i.e. $\det M = 1$, hence is non-singular, an inverse exists. For the following stability result only the rotation R is crucial. The scaling B gives an additional degree of freedom for performance design aspects.

For the following consideration the closed loop system is decomposed into the negative feedback interconnection of subsystem h_2 containing plant, time delay operators and transformation, and the controller h_c, see Fig. 1. The subsystem h_2 can be shown to be IF-OFP. Moreover, the following theorem gives necessary and sufficient conditions for the *exact* preservation of the plant IF-OFP properties δ_p, ε_p through the network to the subsystem h_2 independent of the time delay.

Theorem 1: The subsystem h_2 is IF-OFP with δ_p, ε_p if and only if the following holds

$$\cot 2\theta = \varepsilon_p b - \frac{\delta_p}{b}, \tag{6}$$

and

$$\alpha(\theta) \geq 0. \tag{7}$$

The second condition (7) merely defines one of the two existing solutions θ of (6) in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$, see Lemma 2. As this solution, denoted by $\theta^\star$, exists for each $b > 0$, this scaling parameter can be chosen freely to meet performance requirements. Crucial point of the proposed approach is that the input-output transformation transforms the IF-OFP plant h_p into a finite gain $\mathscr{L}_2$ stable subsystem h_1 such that

$$\|v_{r,t}\| = \|h_1(u_{r,t})_t\| \leq \gamma_{h_1}\|u_{r,t}\| \quad \forall t; \quad \gamma_{h_1}^2 = \frac{\alpha(\theta^\star)}{\beta(\theta^\star)}, \tag{8}$$

holds where $\beta(\theta^\star)$ is defined analogously to (4), $\alpha(\theta^\star) \geq 0$ (7) and $\beta(\theta^\star) > 0$, see Lemma 2. The derivation of the gain γ_{h_1} can be found in the proof of Theorem 1. A constant time delay operator does not alter the signals $\mathscr{L}_2$ gain, i.e. its energetic properties, during transmission, since it has a $\mathscr{L}_2$ gain equal to one $\gamma_{T_1} = \gamma_{T_2} = 1$. The inverse transformation M^{-1} is a bijection as well, and therefore the exact energetic plant properties are recovered at the subsystem h_2.

From this result it is straightforward to conclude finite gain $\mathscr{L}_2$ stability of the closed loop system.

Corollary 1: The closed loop system with the transformation (5) and the transformation parameters satisfying Theorem 1 is delay-independent finite gain $\mathscr{L}_2$ stable.

Thus, a bounded input $w \in \mathscr{L}_{2e}$ implies a bounded output $y_p \in \mathscr{L}_{2e}$. Moreover, it implies all signals being bounded $e, u_c, y_c, u_l, v_l, u_r, v_r, u_p, y_p \in \mathscr{L}_{2e}$, as shown in the proof. As an important result, the feedback interconnection of any controller-plant pair satisfying the finite gain $\mathscr{L}_2$ condition from Proposition 1 *without time delay* is finite gain $\mathscr{L}_2$ stable for *arbitrary large time delay* by using the proposed transformation.

An interesting viewpoint gives the interpretation of Theorem 1 from a small gain perspective. Therefore the closed loop system is decomposed into the subsystems h_1, h_3 h_{T_1}, and h_{T_2} where the *transmitted* signals u_l, u_r, v_r, v_l act as inputs and outputs. The open loop system

$$h_{OL} = h_3 \circ h_{T_1} \circ h_1 \circ h_{T_2} \tag{9}$$

is considered with $w = 0$, see Fig. 1. From the previously stated boundedness of all inputs and outputs it is clear that there exists a finite $\mathscr{L}_2$ gain $\gamma_{OL} < \infty$ such that

$$\|h_{OL}(v_{l,t})_t\| \leq \gamma_{OL}\|v_{l,t}\| \quad \forall t, \tag{10}$$

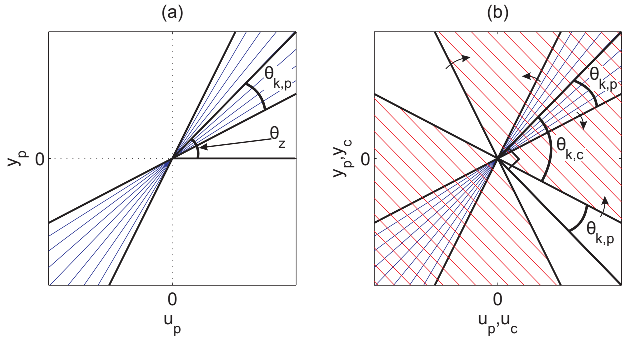

Fig. 2. (a) The sector of an IF-OFP plant. (b) The sector of the plant and the controller satisfying Proposition 1.

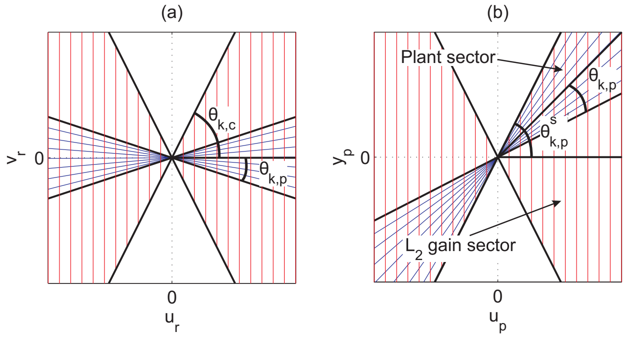

Fig. 3. (a) Finite gain $\mathscr{L}_2$ stable system after applying input-output transformation to the plant and controller from Fig. 2 (b). (b) eqivalent $\mathscr{L}_2$ gain sector of an IF-OFP system.

with $\gamma_{OL} = \gamma_{h_3}\gamma_{T_1}\gamma_{h_1}\gamma_{T_2} = \gamma_{h_3}\gamma_{h_1}$. In fact, the system satisfies the small gain theorem in transformed coordinates.

Corollary 2: The open loop system h_{OL} has a $\mathscr{L}_2$ gain $\gamma_{OL} < 1$.

As shown in the proof, the subsystem h_3 has a $\mathscr{L}_2$ gain γ_{h_3} with $\gamma_{h_3}^2 < \beta(\theta^\star)/\alpha(\theta^\star)$ as long as Proposition 1 is strictly satisfied. Hence, with (8) the small gain property in the communicated variables can be deduced. In fact, with equality in Proposition 1, i.e. marginal stability, also the open loop gain becomes $\gamma_{OL} = 1$. The $\mathscr{L}_2$ gains of the transformed subsystems h_1 and h_3 depend on the IF-OFP properties of plant and controller $\gamma_{h_1} = \gamma_{h_1}(\delta_p, \varepsilon_p)$ and $\gamma_{h_3} = \gamma_{h_3}(\delta_c, \varepsilon_c)$. More conservative, i.e. higher values of δ_p, ε_p and δ_c, ε_c in Proposition 1 result in a smaller open loop gain, hence higher stability reserve. Note, that the small gain theorem is only satisfied for the mappings with the communicated (transformed) variables u_l, u_r, v_r, v_l as input/output, but not for the mappings with the (original) control variables e, y_c, u_p, y_p.

Remark 1: In case of unstable plants the proposed approach locally pre-stabilizes in an input-output sense, by the righthand input-output transformation. This becomes clear from (8), where every IF-OFP plant h_p described by (2) results in a finite gain $\mathscr{L}_2$ stable system h_1 after the right hand transformation.

Remark 2: The proposed approach is straightforward to apply for multi-input-multi-output plants, however, with the restriction that plant inputs and outputs have the same dimension m. Current investigations include the extension to systems with different input-output dimension.

Remark 3: For passive plants, i.e. with $\delta = \varepsilon = 0$ in (2), the proposed input-output transformation $\theta = \frac{\pi}{4}$ is equivalent the well-known scattering transformation [11], [16] widely used for the stabilization of teleoperation systems. Hence, the scattering transformation is a special case of the proposed input-output transformation.

Remark 4: Instead of the time delay operator any other causal operator with a $\mathscr{L}_2$ gain $\gamma \leq 1$ can be inserted without affecting the above stability result. In fact, many scattering based approaches to cope with communication effects such as time-varying delay [17], [18] and packet loss [19], [20] are based on the small gain principle within the scattering variable domain. These approaches are straightforward to apply here.

A. Conic Sectors Interpretation

Conic sectors in the input-output space give a nice geometrical intuition of IF-OFP systems behavior, see e.g. [8], [9]. In a similar manner the input-output transformation can be interpreted as a rotation and a scaling of conic sectors as discussed in the following. For ease of discussion a memory-less IF-OFP system is considered as plant even though stability related notions are futile in this case. The IF-OFP property (2) holds instantaneously, i.e.

$$u_p y_p \geq \delta_p u_p^2 + \varepsilon_p y_p^2, \quad \forall t, \quad (\delta_p, \varepsilon_p) \in \Omega. \tag{11}$$

Geometrically, this equation describes a conic sector in the u_p-y_p-plane (including its in the origin mirrored part), where the system input-output is instantaneously confined to, see [8], [9] for a similar consideration. The following results are mainly derived from (11) by parameterizing the plant input and output in polar coordinates $u_p(t) = r_p(t)\cos\theta_p(t), \quad y_p(t) = r_p(t)\sin\theta_p(t)$ and some straightforward mathematical manipulation.

A conic sector in the plane is described by its center-line angle and its apex angle. The center-line angle θ_z for the conic sector defined by (11) is the solution of

$$\cot 2\theta_z = \varepsilon_p - \delta_p, \tag{12}$$

in the interval $[0, \frac{\pi}{2}]$, and the apex angle $2\theta_{k,p}$ the solution of

$$\cos 2\theta_{k,p} = \frac{\varepsilon_p + \delta_p}{\sqrt{(1 - 4\delta_p\varepsilon_p) + (\varepsilon_p + \delta_p)^2}},$$

with $\theta_{k,p} \in [0, \frac{\pi}{2})$. At each time instant t the input-output of a memory-less system lies within the conic sector $\theta_p(t) \in [\theta_z - \theta_{k,p}, \theta_z + \theta_{k,p}]$ or its mirrored counterpart, see Fig 2 (a) for a visualization.

1) Sector Interpretation of Proposition 1: Given the plant sector by (11) the finite gain $\mathscr{L}_2$ stability condition determines the allowable controller sector. Using a similar technique as in proof of Corollary 2, the allowable controller sector is derived to be $\theta_c(t) \in (\theta_z - \bar{\theta}_{k,p}, \theta_z + \bar{\theta}_{k,p})$ where $\bar{\theta}_{k,p} = \frac{\pi}{2} - \theta_{k,p}$. Note, that due to the strict inequality in Proposition 1 the controller is confined to an open set in a sector with the same center-line as the plant, and complementary angle with respect to 180^o. The larger the sector of the plant is, the smaller the allowable sector for the controller. The plant sector together with the for stability allowable controller sector is visualized in Fig. 2 (b).

2) Input-Output-Transformation: By the input-output transformation the IF-OFP plant with input u_p and output y_p is transformed into a finite gain $\mathcal{L}_2$ stable subsystem h_1 with input u_r and output v_r. Observe that the center-line angle for the IF-OFP plant given by (12) is equal to the rotation angle $\theta^\star$ derived from Theorem 1 for $b = 1$, i.e. without scaling $B = I_2$. Considering a scaling $b \neq 1$ in the transformation the center-line of the IF-OFP plant and controller sector changes to $\cot 2\theta_z = \varepsilon_p b - \delta_p/b$. This is exactly the rotation angle given by Theorem 1 for the general case with scaling. Thus, by the transformation the sector of the plant is rotated such that the sector of the subsystems h_1 has a center-line angle of $\theta_z = 0$. This, however, is exactly the conic sector representation for finite gain $\mathcal{L}_2$ stability, e.g. for the plant side in the transformed coordinates $\|v_{r,t}\| \leq \gamma_{h_1}\|u_{r,t}\|$. The apex angles $2\theta_{k,p}$, $2\theta_{k,c}$ of the plant and of the also rotated allowable controller sector are invariant to the rotation though may have changed due to scaling to $2\theta'_{k,p}$, $2\theta'_{k,c}$, they are related to the $\mathcal{L}_2$ gain by $\tan\theta'_{k,p} = \gamma_{h_1}$ and $\tan\theta'_{k,c} = 1/\gamma_{h_1}$. The allowable controller area thus, expresses the small gain theorem of the open loop with the "rotated" subsystems h_1 and h_3 as has been shown also in Corollary 2. The rotation of the IF-OFP plant and controller from Fig. 2(b) to a finite gain $\mathcal{L}_2$ stable system is visualized in Fig. 3(a).

For comparison the classical small gain approach is discussed. For simplicity $b = 1$ is considered. Clearly, the IF-OFP plant from Fig. 2 (a) can also be represented by an *enlarged* conic sector symmetric to the u_p axis, as shown in Fig. 3 (b). For the open loop gain $\gamma_p^s\gamma_c^s = \tan(\theta^s_{k,p})\tan(\theta^s_{k,c}) < 1$ has to hold, where $|2\theta^s_{k,p}| \geq |2\theta_{k,p}|$ is the apex angle of the enlarged conic sector for the plant. Accordingly, the stability allowable controller sector with apex angle $|2\theta^s_{k,c}| \leq |2\theta_{k,c}|$ is smaller than with the transformation approach, i.e. is more conservative.

The main idea of the proposed approach can be summarized as rotating the plant and controller conic sectors to achieve a non-conservative $\mathcal{L}_2$ gain representation in the communicated signals compared to the classical small gain approach. Arbitrarily large constant time delay does not alter this representation.

Note, however, that Corollary 1 gives only a sufficient condition for finite gain $\mathcal{L}_2$ stability as it relies on the sufficient stability condition from Proposition 1. This can be expected as only very few knowledge of the plant and controller input-output relation is required. In the following LTI systems with *known* transfer functions are considered for plant and controller to obtain a necessary and sufficient condition for delay-independent stability.

B. Stronger Stability Condition for Known LTI Systems

The LTI plant and controller are described by the transfer functions

$$G_p(s) = \frac{Y_p(s)}{U_p(s)}, \qquad G_c(s) = \frac{Y_c(s)}{E(s)}.$$

respectively, where $Y_p(s)$ and $U_p(s)$ represent the Laplace transformations of the plant output $y_p(t)$ and input $u_p(t)$, and $Y_c(s)$ and $U_c(s)$ the Laplace transformations of the controller output $y_c(t)$ and input $e(t)$. Accordingly, $G_1(s), G_3(s)$ denote the transfer functions describing the h_1 and h_3, respectively. The open loop transfer function similar to (9) but without the time delays T_1, T_2, i.e. $G_{OL}(s) = G_1(s)G_3(s)$, is given by

$$G_{OL} = \frac{b^2G_c + G_p - b\tan(\theta) - b\cot(\theta)G_cG_p}{b^2G_c + G_p + b\cot(\theta) + b\tan(\theta)G_cG_p}. \tag{13}$$

where the Laplace variable s is dropped for convenience of notation. Delay-independent stability can be guaranteed only if the small gain theorem is satisfied.

Corollary 3: The linear time-invariant closed loop system is asymptotically stable independent of constant time delay if and only if $|G_{OL}|^\infty < 1$.

Proof: The proof is straightforward by considering the full open loop transfer function, i.e. including the forward and backward time delay with $T = T_1 + T_2$, which clearly satisfies the small gain theorem as

$$|G_{OL}e^{-j\omega T}|^\infty \leq |G_{OL}|^\infty |e^{-j\omega T}|^\infty = |G_{OL}|^\infty < 1\ . \qquad \blacksquare$$

This condition is necessary and sufficient. Applying Theorem 1 leads to the more conservative stability result $\gamma_{h_1}\gamma_{h_3} = |G_1|^\infty |G_3|^\infty < 1$. The conservativeness comes from the fact that most generally $|G_1G_3|^\infty \leq |G_1|^\infty |G_3|^\infty$ holds with strict inequality. Equality is given only if the maximum magnitude of G_1 and G_3 appears at the same frequency $\omega_{max} = \arg\sup_\omega \bar{\sigma}[G_1] = \arg\sup_\omega \bar{\sigma}[G_3]$ with $\bar{\sigma}$ the maximum singular value.

Under the restriction of Corollary 3, the controller and the transformation can conjointly be designed in the LTI case with known transfer functions; knowledge of the time delay value is not required.

V. PERFORMANCE ISSUES

In the following some performance issues as the steady state behavior, and sensitivity to time delay uncertainty are briefly discussed for LTI plants and controllers. The closed loop transfer function $G(s)$ from the reference input W to the plant output Y_p is computed by the transformation equations (3) to be,

$$G(s) = \frac{Y_p(s)}{W(s)} = G_0(s)G_{tr}(s)e^{-sT_1} \tag{14}$$

where

$$G_{tr}(s) = \frac{1 - G_{OL}(s)}{1 - G_{OL}(s)e^{-sT}}. \tag{15}$$

with G_{OL} from (13) and

$$G_0(s) = \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)}.$$

The system can be seen as a series connection of the standard closed loop system G_0 without time delay and transformation and G_{tr} describing the influence of the time delay and the input-output transformation. be aggressively designed without considering delay in the loop.

1) Steady State Behavior: The steady state behavior with transformation and time delay is equivalent to the steady state behavior without transformation and without time delay as easily derivable by setting $s = 0$ in (14) and (15) resulting in $G(0) = G_0(0)$. For the nonlinear case this can be observed from the steady state condition $s_l = s_r$, hence $z_c = M^{-1}s_l = M^{-1}s_r = M^{-1}Mz_p = z_p$.

In terms of steady state error the proposed approach clearly outperforms the standard small gain approach which requires $|G_c(j\omega)G_p(j\omega)| < 1, \omega \geq 0$, i.e. free integrators in the open loop are not allowed. This leads to a rather large steady state error, e.g. $|e(t)|_{t\to\infty} > \frac{1}{2}|w|$ for a reference step input w. In the proposed approach free integrators in plant or controller do not necessarily appear as free integrators in G_{OL} (13). As a result delay-independent stability can still be guaranteed by Corollary 3. Further, since the steady state of the closed loop system is not affected by the transformation, an integrator in the plant or the controller will guarantee zero steady state error. This can be easily demonstrated using examples, e.g. $G_p(s) = \frac{1}{s+1}, G_c(s) = \frac{s+1}{s(s+10)}, b = 1, \theta = 30^o$ satisfies Corollary 3 and has zero steady state error.

2) Sensitivity with Respect to Time Delay Uncertainty: The proposed approach guarantees delay-independent stability, in the following we show that by appropriate design also the tracking behavior is rather insensitive to time delay uncertainty. Low sensitivity to time delay uncertainty means that a similar performance can be guaranteed in a large range of time delay values. The sensitivity of the closed loop system input-output behavior (14) with respect to time delay is given by

$$S_T^{G^*} = \frac{T}{G^*}\frac{dG^*}{dT} = sTe^{-sT}\frac{G_{OL}}{1 - G_{OL}e^{-sT}},$$

where $G^*(s) = G_0(s)G_{tr}(s)$ is the transfer function (14) without the purely time shifting part e^{-sT_1}. The sensitivity becomes low for small G_{OL}. Note that this does not contradict the small gain stability requirement for G_{OL} from Corollary 3. Insensitivity $S_T^{G^*} = 0$ can be achieved using a proportional controller $G_c(s) = \frac{1}{b}\tan\theta$, independently of the plant. This follows straightforward from substituting G_c in (13) resulting in $G_{OL} = 0 \Rightarrow S_T^{G^*} = 0 \Rightarrow G_{tr}(s) = 1$. The closed loop transfer function (14) reduces to $G(s) = G_0(s)e^{-sT_1}$ with the time shifting part having no effect on the transient performance. However, a proportional controller usually does not meet the performance requirements. Generally, a compromise should be made between performance and insensitivity with respect to time delay uncertainty.

3) Zero Time Delay Case: As the time delay reduces to zero, i.e. $T_1 = T_2 = T = 0$, the system reduces to that without input-output transformation, i.e. $G(s) = G_0(s)$ as straightforward computable from (14) and (15). The statement holds as well for the general nonlinear case. This is interesting as the controller can compared to the standard small gain approach be rather aggressively designed without considering time delay. For zero time delay “nominal” performance is recovered. Together with low sensitivity this means that performance deteriorates gracefully for increasing time delay.

In summary, the proposed approach shows significant advantages over the standard small gain approach. In fact, even delay dependent input-output approaches are outperformed as shown in a forthcoming paper [21] in theory and experiments. Here we demonstrate its efficacy by a numerical example.

VI. NUMERICAL EXAMPLE

The goal of this example is to show stability and the low sensitivity to time delay of the closed loop system with the proposed transformation approach. The controller is designed a-priori without considering the constant time delay. The parameters b, θ are chosen afterwards according to Corollary 3.

A stable LTI plant is considered with the transfer function

$$G_p(s) = \frac{1}{(0.5s+1)(0.1s+1)}.$$

A PI controller, heuristically tuned to give without time delay a compromise between overshoot and rise time and zero steady state error, is used

$$G_c(s) = \frac{6(0.7s+1)}{s}.$$

In order to achieve low time delay sensitivity in the low frequency range the rotation angle θ is chosen as a function of the scaling parameter b according to

$$S_T^{G^*}(0) = G_{OL}(0) = 0 \Rightarrow \theta = \cot^{-1}(b), \qquad (16)$$

and the optimization problem

$$\min_b |G_{OL}(j\omega)|^\infty,$$

is solved numerically for the intervals $\omega \in [10^{-2}, 10^3]$ and $b \in [10^{-2}, 10^2]$ by carpet search using the `fminbnd` function of the MATLAB optimization toolbox. The result of the optimization is $b = 0.4852$ and by (16) $\theta = 1.11\,(64.12^\circ)$. It is straightforward to show that $|G_{OL}|^\infty < 1$ holds, i.e. the system is delay-independently stable. The step responses for different values of the round trip time delay $T \in \{20, 40, 80, 200\}$ ms in Fig. 4 show a graceful performance degradation with increasing time delay when the proposed approach is applied. As expected, without transformation large oscillations occur with increasing time delay; the system is unstable for $T = 200$ ms.

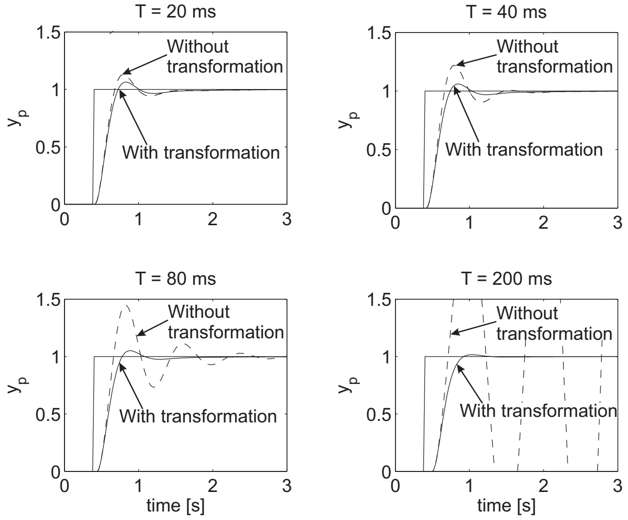

Fig. 4. Step response of the system with and without the transformation for various round trip time delay values.

VII. CONCLUSIONS

This paper presents a novel input-output approach to stabilize networked control systems in the presence of arbitrarily large constant time delay. A linear combination of plant and controller input and output is transmitted over the communication channel instead of the direct plant and controller output. It is applicable to all input-feedforward-output-feedback passive plants including time-variant and non-linear systems. A sufficient condition for finite gain $\mathscr{L}_2$ stability is derived for the general non-linear case with largely unknown model; a necessary and sufficient stability condition for LTI systems with known transfer functions. The proposed approach allows non-conservative controller design without considering time delay in the loop resulting in superior tracking performance. Due to the low sensitivity to time delay the performance remains good even for high time delay values. In this sense the proposed approach outperforms many other known time-delay input-output approaches. Future research addresses optimal controller design, the investigation of more general transformations and, the varying time delay and packet loss problem.

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APPENDIX

Proof of Lemma 1: If $\delta,\varepsilon\in\bar{\Omega}=\Omega_3\cup\Omega_4$ with $\Omega_3=\{\delta,\varepsilon\in\mathbb{R}|\delta\varepsilon\ge 1/4;\varepsilon<0\}$, and $\Omega_4=\{\delta,\varepsilon\in\mathbb{R}|\delta\varepsilon>1/4;\varepsilon>0\}$, degenerate cases occur. In case $\delta,\varepsilon\in\Omega_3$ multiplying (2) with $\varepsilon<0$ and taking the square complement it follows

$$\lambda(t)=\|\varepsilon y_t-\frac{u_t}{2}\|^2+(\delta\varepsilon-1/4)\|u_t\|^2\ge 0,\quad\forall t$$

which is satisfied for any pair $u(\tau),y(\tau)$ since $(\delta\varepsilon-1/4)\ge 0$ imposing thus no restriction to the system input-output behavior. Analogously, in case $\delta,\varepsilon\in\Omega_4$ the reformulation of (2) leads to $\lambda(t)\le 0$ which can be satisfied only for $u(\tau)=0$ since $(\delta\varepsilon-1/4)>0$. ■

Proof of Lemma 2: If $(\delta,\varepsilon)\in\Omega_1$, for the two angles $\theta_i, i\in\{1,2\}$ it can be shown that $\alpha(\theta_i)\beta(\theta_i)=\frac{1}{4}-\delta\varepsilon>0$ meaning that α,β have always the same sign for each angle, and furthermore $\alpha(\theta_2)=-\beta(\theta_1),\beta(\theta_2)=-\alpha(\theta_1)$ meaning that $\alpha(\theta_i),\beta(\theta_i)$ have always different signs for the two angle solutions $\theta_i, i\in\{1,2\}$. Combining the above the first part of the lemma is proved.

If $(\delta,\varepsilon)\in\Omega_2$ like before we get $\alpha(\theta_i)\beta(\theta_i)=\frac{1}{4}-\delta\varepsilon=0$ meaning that some of α,β are zero and furthermore $\beta(\theta_i)=\alpha(\theta_i)+\delta+\varepsilon\Rightarrow\beta(\theta_i)>\alpha(\theta_i)$, thus $\alpha(\theta_i)=0,\beta(\theta_i)>0$. ■

Proof of Theorem 1: To be shown is that the energetic input-output behavior, i.e. IF-OFP property with δ_p, ε_p, of the plant is inherited to the subsystem h_2, i.e.

$$\langle y_c,u_c\rangle_t\ge\delta_p\|y_{c,t}\|^2+\varepsilon_p\|u_{c,t}\|^2. \tag{17}$$

(*sufficiency*) Rewriting the IF-OFP condition (2) for the plant in terms of the transmitted variables s_r gives

$$\int_0^t s_r^T M^{-T}SM^{-1}s_r\mathrm{d}\tau\ge\int_0^t s_r^T M^{-T}DM^{-1}s_r\mathrm{d}\tau \tag{18}$$

with

$$S=\begin{bmatrix}0&\frac{1}{2}\\ \frac{1}{2}&0\end{bmatrix};D=\begin{bmatrix}\delta_p&0\\0&\varepsilon_p\end{bmatrix}.$$

Subtracting the lefthand side from the righthand side we get

$$\int_0^t s_r^T\{c_{i,j}\}s_r\mathrm{d}\tau\ge 0,\quad i,j\in\{1,2\}, \tag{19}$$

with the matrix $\{c_{i,j}\}$

$$c_{1,1}=\alpha(\theta);$$
$$c_{1,2}=c_{2,1}=\frac{1}{2}\cos 2\theta-(\varepsilon_p b-\frac{\delta_p}{b})\frac{1}{2}\sin 2\theta=\zeta(\theta);$$
$$c_{2,2}=-\beta(\theta)=-\alpha(\theta)-\frac{\delta_p}{b}-\varepsilon_p b.$$

By inserting (6) into $c_{1,2}$, $c_{2,1}$ we can rewrite (19)

$$\alpha(\theta)\|u_{r,t}\|^2-\beta(\theta)\|v_{r,t}\|^2\ge 0,$$

hence the cross terms in the quadratic form are cancelled.

According to Lemma 1 $\frac{\delta_p}{b}\varepsilon_p b=\delta_p\varepsilon_p\in\Omega$, hence we can choose $\theta_i\in[-\frac{\pi}{2},\frac{\pi}{2}]$ such that $\alpha\ge 0,\beta>0$, see Lemma 2. Thus, the subsystem h_1, see Fig. 1, is finite gain $\mathscr{L}_2$ stable with

$$\|v_{r,t}\|=\|h_1(u_{r,t})_t\|\le\gamma_{h_1}\|u_{r,t}\|\quad\forall t\quad\text{with}\quad\gamma_{h_1}^2=\frac{\alpha(\theta)}{\beta(\theta)}.$$

Taking further into account that the constant time delay operator has an $\mathscr{L}_2$ gain equal to one and using the assumption that $u_l(t)=0\quad\forall t\in[-T_1,0]$ and $v_r(t)=0$ $\forall t\in[-T_2,0]$, we may state

$$\|u_{r,t}\|^2\le\|u_{l,t}\|^2,$$
$$\|v_{l,t}\|^2\le\|v_{r,t}\|^2,\quad\forall t>0.$$

It follows that

$$\alpha(\theta)\|u_{l,t}\|^2 - \beta(\theta)\|v_{l,t}\|^2 \geq 0.$$

Analogously to (18) we may rewrite the latter equation as

$$\int_0^t s_l^T M^{-T} S M^{-1} s_l \mathrm{d}\tau \geq \int_0^t s_l^T M^{-T} D M^{-1} s_l \mathrm{d}\tau \qquad (20)$$

which expressed in the variables y_c, u_c is nothing else than (17). For *necessity* it only has to be shown that without this cancellation the time delay alters the energetic behavior of the subsystem h_1. This is straightforward to show for a counter example such as $y_p(t) = k \cdot u_p(t)$. ■

Proof of Corollary 1: We have to show that bounded input $w \in \mathscr{L}_{2e}$ implies bounded output $y_p \in \mathscr{L}_{2e}$. By applying Proposition 1 to the closed loop system decomposed into subsystems h_2 and h_c it is straightforward that also the signals $u_c, y_c, e \in \mathscr{L}_{2e}$. It remains to show that this implies the plant output $y_p \in \mathscr{L}_{2e}$. Since u_l, v_l are linear combinations of u_c, y_c we have $u_c, y_c \in \mathscr{L}_{2e} \Rightarrow u_l, v_l \in \mathscr{L}_{2e}$. The constant time delay operator is a finite gain $\mathscr{L}_2$ stable operator so $u_l, v_l \in \mathscr{L}_{2e} \Rightarrow u_r, v_r \in \mathscr{L}_{2e}$. Since again u_p, y_p are a linear transformation of u_r, v_r, we have that $u_r, v_r \in \mathscr{L}_{2e} \Rightarrow u_p, y_p \in \mathscr{L}_{2e}$, i.e. there exists a $\gamma < \infty$ such that $\|y_{p,t}\| \leq \gamma\|w_t\|$ holds $\forall t$. Assuming the plant output to be unbounded, i.e. $y_p \notin \mathscr{L}_{2e}$, results with the same arguments as above in a contradiction to the assumption $w \in \mathscr{L}_{2e}$. ■

Proof of Corollary 2: For the subsystem h_{OL} it is straightforward to show that

$$\|h_{OL}(v_{l,t})_t\| \leq \gamma_{OL}\|v_{l,t}\|$$

with $\gamma_{h_{OL}} = \gamma_{h_3}\gamma_{T_1}\gamma_{h_1}\gamma_{T_2} = \gamma_{h_3}\gamma_{h_1}$ since for the time delay operators $\gamma_{T_1} = \gamma_{T_2} = 1$ holds. It remains to show that $\gamma_{h_3}\gamma_{h_1} < 1$. Substituting the transformation equations (3) in (2) for the plant and using inequality $kuy \leq |k|\frac{1}{2}(u^2 + y^2), k \in \mathbb{R}$, an upper bound for the $\mathscr{L}_2$ gain of the subsystems h_1 is computed by

$$\|v_{r,t}\| = \|h_1(u_{r,t})_t\| \leq \gamma_{h_1}\|u_{r,t}\|, \quad \text{with } \gamma_{h_1}^2 \leq \frac{\alpha(\theta) + |\zeta(\theta)|}{\beta(\theta) - |\zeta(\theta)|}$$

The $\mathscr{L}_2$ gain γ_{h_3} of the subsystems h_3 depends on the values δ_c and ε_c. In order to derive a statement on the open loop gain $\gamma_{h_3}\gamma_{h_1}$, γ_{h_3} is parameterized according Proposition 1 in terms of δ_p, ε_p and a deviation Δ accounting for the strict inequality. Therefore we set

$$\Delta = \min[(\varepsilon_p + \delta_c)b, (\varepsilon_c + \delta_p)/b] > 0.$$

It is straightforward to see that the controller satisfies also (2) with $\delta_c' = -\varepsilon_p + \Delta/b$, $\varepsilon_c' = -\delta_p + \Delta b$ as $\delta_c' \leq \delta_c$, $\varepsilon_c' \leq \varepsilon_c$. For, these values $\delta_c', \varepsilon_c'$ and following the same procedure as before an upper bound for the $\mathscr{L}_2$ gain of the subsystem h_3 is computed

$$\|u_{l,t}\| = \|h_3(v_{l,t})_t\| \leq \gamma_{h_3}\|v_{l,t}\|, \quad \text{with } \gamma_{h_3}^2 \leq \frac{\beta(\theta) + |\zeta(\theta)| - \Delta}{\alpha(\theta) - |\zeta(\theta)| + \Delta}$$

Choosing θ from (6) (7) it follows $\zeta(\theta) = 0$ and thus

$$\gamma_{h_3}^2\gamma_{h_1}^2 \leq \frac{\alpha(\theta)}{\beta(\theta)}\frac{\beta(\theta) - \Delta}{\alpha(\theta) + \Delta} < 1,$$

hence $\gamma_{h_3}\gamma_{h_1} < 1$, and thus $\gamma_{h_{OL}} < 1$. ■

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