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Hybrid Trajectory Planning for a Mechatronic Tilting System

Marion Sobotka, Martin Buss
Institute of Automatic Control Engineering
Technische Universität München
D-80290 München, Germany
marion.sobotka@ei.tum.de, martin.buss@ei.tum.de

***Abstract*— In this paper we discuss two trajectory design approaches for a mechatronic tilting system using a hybrid (discrete-continuous) modeling framework. Tilting means that the mechanical system under consideration rolls passively (unactuated) over one of its edges in contact with the environment. This occurs, for example, for walking machines when the foot tilts over its edge. Hybrid (discrete-continuous) system approaches have a great mechatronics application potential in the area of systems with (multiple) varying contacts between system and environment, e.g. walking systems, cars, hands, etc. The simple model system considered in this paper consists of an actuated arm, which is fixed on a base plate that can tilt over its edges. A mathematical model and two trajectory planning methods using a hybrid dynamical systems approach are proposed and the resulting trajectories are discussed in simulation. Some of the trajectories show sensitive parameter dependence, which is characteristic for chaotic systems.**

I. Introduction

Motion planning for biped or multiped robots does usually not imply tilting around the foot edges [1]. It is assumed that tilting will not occur, if the trajectories are precalculated properly and a higher-level online control ensures stability [2]. But especially for faster moving robots, tilting could be part of a planned trajectory to improve the performance, e.g. by mimicking the foot roll motion of humans. Recently, some humanoid robots include an actuator in the foot to enable motions similar to the human toe, see e.g. [3].

Tilting systems in general have distinguishable discrete states, as at least "tilted" and "not tilted". Therefore a hybrid dynamical system description is required. The class of hybrid dynamical systems cannot be described with a purely continuous neither a purely discrete model. The hybrid state model proposed in [4] uses mixed continuous and discrete states; discrete states are typically integer-valued, without loss of generality.

Due to the nonlinear, hybrid and not fully actuated character of tilting systems, precalculation of trajectories is a challenge. The event time from stable contact to tilting is determined by the Zero Moment Point condition [5], depending nonlinearly on states and control inputs.

We discuss a mechanical system that is simple enough to allow straightforward generation of trajectories. The first presented method is related to an approach by Nakanishi *et al.* [6], who find a control law for the control on ape-like brachiation trajectories by determining parameters of predefined motions, such that the resulting motion fulfills e.g. symmetry properties. The second method we discuss is related to an approach by Bühler *et al.* [7]. They discussed the control of a juggling robot with a mirror control law, where unstable orbits are stabilized by an energy conservation approach. We use a similar control architecture and confirm the occurrence of *chaotic behavior* for certain parameters. Our system combines the problems of juggling, where collisions occur, with the problems of brachiation, where the robot is underatuated. In addition we have several contact situations with complicated transition conditions between them. Chaotic behavior for a hybrid system was also shown by Engell *et al.* for a hybrid water tank system, see [8].

II. Hybrid Modeling

We consider a simple mechanical system that allows for tilting and has the advantage that the dynamical equations can still be derived analytically. It consists of an actuated arm that is fixed on a ground plate as shown in Fig. 1.

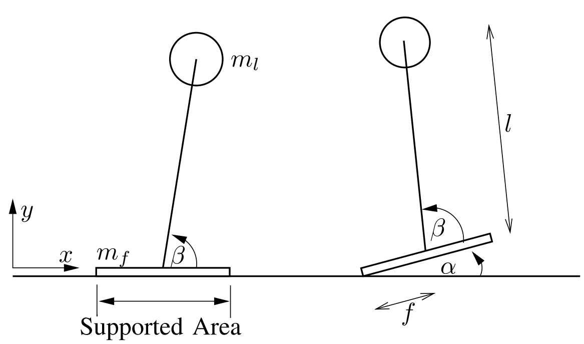

Fig. 1. Geometry of the Tilting Robot. Illustration of the definition of lengths, masses, angles, supported area and the coordinate system.

Tilting is defined as a free rotational movement around one foot edge and occurs when the Zero Moment Point (ZMP) [5] crosses the boundary of the supported area, which is the region covered by the foot plate. The ZMP is a point on ground level, where the ground reaction force acts to compensate all horizontal moments thus keeping the system in balance. A hybrid modeling framework [4] has to be chosen, as the dynamical description for tilting differs from the dynamical description for a stable contact situation and transitions are non-smooth due to collisions. An overview of the transition structure between the three possible contact situations is given in Fig. 2.

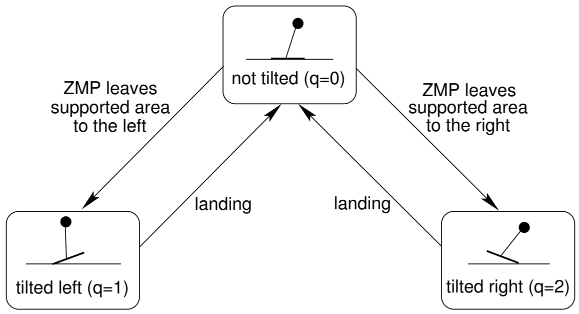

Fig. 2. Transition Structure between the discrete states. While resting in the contact state "not tilted" an ordinary differential equation describes the continuous motion. When crossing a transition surface (Event: ZMP is leaving the supported area or landing) the differential equation changes and discontinuities in the states occur.

A. Hybrid State Vector

An essential step in modeling hybrid systems with the HDS framework is the adaption of the state vector concept. For a dynamical system, the specification of the state vector for an initial time manifests the behavior of the system for all times. The same property is realized for hybrid systems, through the extension of the continuous state vector $\boldsymbol{x} = (\begin{array}{cccc} \alpha & \beta & \dot{\alpha} & \dot{\beta} \end{array})^T$ by a discrete variable q, that stands for the actual discrete state of the system where $q = 0$, when the system is not tilted, $q = 1$ when it is tilted left and $q = 2$, when it is tilted right. In the following α is referred to as tilt angle and β is referred to as actuated angle. Accordingly, the hybrid system state vector is

$$\boldsymbol{\zeta} = (\begin{array}{ccccc} \alpha & \beta & \dot{\alpha} & \dot{\beta} & q \end{array})^T$$

The state trajectories are mainly continuous in correspondence with the mainly continuous character of the system. Discontinuities occur only, when the system starts tilting or stops tilting. The mathematical conditions for a discrete switch are thus modeled as surfaces in state space and the state trajectory is constantly checked on a crossing of one of these surfaces. A transition on the one hand allows for a discontinuity in the continuous state vector part, as it occurs for example when collisions are modeled, on the other hand a reset of the discrete state variable q is possible, e.g. when different dynamical equations are valid in different state space regions. Thus a differential description for any of the three contact situations "not tilted", "left tilted" and "right tilted" has to be derived. In addition mathematical conditions for the transition between the contact situations are needed including a formal description of the discontinuous behavior at transition time.

B. Continuous Dynamics

The equations of motion for all of the three contact scenarios are based on an Euler-Lagrange approach [9]. The positions $\boldsymbol{r}_f$ and $\boldsymbol{r}_l$ of the centers of mass of the links are expressed in terms of the orientation angles α and β and of the spatial position of a reference point ξ and η. Here the left foot edge is chosen as reference point and the masses are assumed to be concentrated in one point.

$$\begin{aligned} \boldsymbol{r}_f &= \begin{pmatrix} r_{fx} \\ r_{fy} \end{pmatrix} &= \begin{pmatrix} \xi + f\cos(\alpha) \\ \eta + f\sin(\alpha) \end{pmatrix} \\ \boldsymbol{r}_l &= \begin{pmatrix} r_{lx} \\ r_{ly} \end{pmatrix} &= \begin{pmatrix} \xi + f\cos(\alpha) + l\cos(\alpha+\beta) \\ \eta + f\sin(\alpha) + l\sin(\alpha+\beta) \end{pmatrix} \end{aligned}$$

The length of the half foot plate is f and the mass of the footplate is m_f, the length of the arm is l and the mass m_l of the arm is assumed to be concentrated at the tip. The constant g is the gravitational acceleration. The Lagrangian Function L is then defined as

$$L = \frac{1}{2} m_f \|\dot{\boldsymbol{r}}_f\|^2 + \frac{1}{2} m_l \|\dot{\boldsymbol{r}}_l\|^2 - m_f g r_{fy} - m_l g r_{ly}$$

and a set of four equations of motion of second order is determined by the Euler-Lagrange Equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\varphi}} - \frac{\partial L}{\partial \varphi} = \tau_\varphi$$

where $\varphi = \alpha, \beta, \xi, \eta$ and $\tau_\varphi = 0$ for $\varphi = \alpha, \xi, \eta$ because only β is directly actuated. In the following τ_β is abbreviated with τ.

The actual ground contact situation is accounted for by introducing appropriate algebraic constraint equations $\boldsymbol{g}_i(\alpha, \beta, \xi, \eta) = 0$. Three configurations are possible: If the system is neither tilted right nor tilted left the constraints are

$$\boldsymbol{g}_0(\alpha, \beta, \xi, \eta) = \begin{pmatrix} \alpha \\ \xi \\ \eta \end{pmatrix},$$

if the systems tilts to the left the constraints are

$$\boldsymbol{g}_1(\alpha, \beta, \xi, \eta) = \begin{pmatrix} \xi \\ \eta \end{pmatrix},$$

and if the robot tilts to the right the constraints are

$$\boldsymbol{g}_2(\alpha, \beta, \xi, \eta) = \begin{pmatrix} \xi + 2f\cos(\alpha) \\ \eta + 2f\sin(\alpha) \end{pmatrix}$$

The algebraic constraints are combined with the differential equations through the introduction of Lagrangian multipliers [9], which eliminates several degrees of freedom, here in particular the equations of motion for ξ and η. In exchange expressions for generalized contact forces, such as F_y (the total contact force in y-direction) are delivered, further on needed for the calculation of the ZMP.

For notational convenience and simulation purposes the three resulting reduced systems are transformed to systems of first order differential equations, that we abbreviate as

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_q(\boldsymbol{x}, \tau)$$

where $q = 0, 1, 2$ depending on the side constraint used to derive the equation and in this sense denoting the validity of the equation for the contact situation "left tilted" ($q = 1$), "not tilted" ($q = 0$) and "right tilted" ($q = 2$).

C. Discrete Dynamics

A transition between two contact situations is triggered whenever the hybrid orbit crosses a transition surface $s(\boldsymbol{\zeta}, \tau, t) = 0$. Only then a discontinuity in the state vector is allowed, denoted as $\boldsymbol{\zeta}^- \longrightarrow \boldsymbol{\zeta}^+$. Here the superscript $(\cdot)^-$ marks the value of the state variable immediately before the transition, accordingly $(\cdot)^+$ marks the value of the state variable immediately after the transition.

For the tilting system we have two transition surfaces that depend on the location of the ZMP and two transition surfaces that depend on the tilt angle α. The equation for the ZMP r_z is

$$r_z(\boldsymbol{x}, \tau) = \frac{M_z}{F_y},$$

where M_z, the total moment in z-direction with respect to the origin, and F_y, the total force in y-direction, are calculated from the constraint conditions of the differential equation as mentioned before.

The transition surface from contact situation $q = q_1$ to contact situation $q = q_2$ is denoted by $s_{q_1 q_2}(\boldsymbol{\zeta}, \tau, t) = 0$. The transition surface that models the crossover to "tilted left" from the stable contact situation has to detect zero crossings of the ZMP and therefore is

$$s_{01}(\boldsymbol{\zeta}, \tau, t) = r_z(\boldsymbol{x}, \tau) \quad \boldsymbol{\zeta}^+ = (\boldsymbol{x}^-, 1)^T,$$

where the continuous state parts transition is smooth and q is set 1 to account for the new contact situation. Analogously for "tilting right" the ZMP has to cross $2f$, which is the right edge of the foot plate and $q = 2$ from then on.

$$s_{02}(\boldsymbol{\zeta}, \tau, t) = r_z(\boldsymbol{x}, \tau) - 2f \quad \boldsymbol{\zeta}^+ = (\boldsymbol{x}^-, 2)^T$$

The detection of landing from tilted is realized by observing the tilt angle α.

$$s_{10}(\boldsymbol{\zeta}, \tau, t) = \alpha \quad \boldsymbol{\zeta}^+ = (\boldsymbol{h}(\boldsymbol{x}^-), 0)^T$$

$$s_{20}(\boldsymbol{\zeta}, \tau, t) = \alpha \quad \boldsymbol{\zeta}^+ = (\boldsymbol{h}(\boldsymbol{x}^-), 0)^T$$

Since landing is accompanied by a collision with the ground, it results in a non-smooth state transition denoted by a map $\boldsymbol{h}(\boldsymbol{x})$, that implements a collision model.

D. Numerical Simulation of Hybrid Systems

To simulate the application of a control strategy for hybrid systems the solution of the ordinary differential equation

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_q(\boldsymbol{x}, \tau)$$

has to be determined. After every integration step the transition conditions $s(\boldsymbol{\zeta}, \tau, t) = 0$ are evaluated. If a transition condition turns out to become zero the integration is stopped and restarted after reinitialization of the state variable. Attention has to be paid to meet the transition condition $s(\boldsymbol{\zeta}, \tau, t) = 0$ as exactly as possible. For this reason the "Events" option of the Matlab solver for ordinary differential equations is employed. The solver routine meets zero crossings of the transition conditions very precisely.

III. Trajectories

As a first step towards the understanding of hybrid tilting systems we discuss trajectories that cause frequent tilting of the base plate, but still return to a stable configuration with flat ground contact of the base. In other words we seek for hybrid periodic trajectories that regularly cycle between the discrete states "foot flat on the ground", "foot tilted to the left", "foot flat on the ground" and "foot tilted to the right".

Two approaches are considered. The first approach focuses on a trajectory for the actuated angle $\beta(t)$ and the tilt angle trajectory $\alpha(t)$ is only a consequence from the actuated motion. The second approach focuses on the tilt angle trajectory, whereas now the trajectory for the actuated angle β is the consequence. For simple symmetric hybrid structures it is possible to determine periodic orbits as solutions of a boundary value problem. This is demonstrated in the following subsections.

A. Trajectories with focus on the actuated angle

1) Boundary value problem for the trajectories: To achieve a periodic hybrid trajectory, we constrain the actuated arm upon a predefined periodic trajectory,

$$\beta^d(t) = A \sin(\omega t) + \frac{\pi}{2}$$

with period length T and solve the problem how to determine the parameters A and ω of the predefined motion such that the resulting hybrid orbit is periodic and symmetric concerning left tilting and right tilting. In the derivation it is assumed, that a control strategy is available, that enables exact trajectory following despite of possible disturbances by the collision between foot and ground. The shortcoming through this idealization is discussed later, when a feasible controller is used in a computer simulation.

The hybrid trajectory starts in the discrete state "foot flat on the ground", where $\beta(t = 0) = \pi/2$. As long as the ZMP reaches neither the left edge nor the right edge of the foot plate no tilting occurs. The parameter pairs (A, ω) for which the hybrid orbit does not reach any transition surface can be determined in advance. For any other choice of (A, ω) the system will start tilting and it has to be ensured by a boundary condition that the system returns again to a stable contact situation. The trajectory will first reach the transition surface towards tilting left at a time $t = t_t$, where the tilt angle $\alpha(t_l)$ and the tilt velocity $\dot{\alpha}(t_l)$ are still zero, and form the initial conditions for the boundary value problem. Then for $t > t_t$ the system is in the "tilting left" state. For hybrid periodicity with period length T obviously landing from tilted is essential. For symmetric orbits, $\frac{T}{2} - t_t < t_l < \frac{T}{2} + t_t$ must be true, because only then the ZMP is in the supported area at landing time, see Fig. 3. This prevents immediate tilting to the other direction and guarantees that the initial condition for left tilting mirrors the initial condition for right tilting. The landing time condition constitutes the boundary condition of the boundary value problem. The parameters A and ω of the steering trajectory for β are varied to find a solution that fits the initial condition as well as the boundary condition.

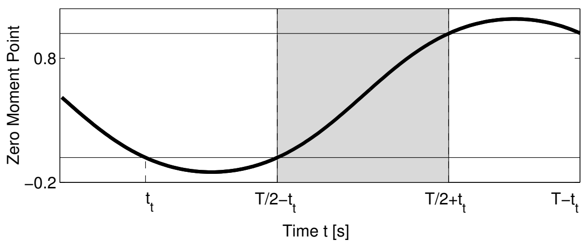

Fig. 3. Zero Moment Point (ZMP) and Landing Time. Due to the periodic trajectory for β, the trajectory of the ZMP is periodic. Only for $\frac{T}{2} - t_t < t_l < \frac{T}{2} + t_t$ the resulting hybrid tilt orbit is symmetric. (Here $f = 0.5$ is chosen, the supported area is between 0 and 1.)

In summary the boundary value problem consists of the parameter-dependent differential equation for α

$$\ddot{\alpha} = F(\alpha, \dot{\alpha}; A, \omega; t),$$

initial conditions at tilting time t_t

$$\alpha(t_t) = 0$$

$$\dot{\alpha}(t_t) = 0$$

and a boundary condition at a desired landing time t_l

$$\alpha(t_l) = 0.$$

Note that the boundary value problem needs only one parameter to be solvable. Thus an infinite set of (A, ω) pairs for every landing time t_l can be determined as the solution. Solutions pairs for selected landing times t_l are shown in Fig. 4. The calculations here and in the following are performed with $m_f = 1$, $m_l = 1$, $l = 2$ and $f = 0.5$, thus the supported area is the interval $[0, 1]$.

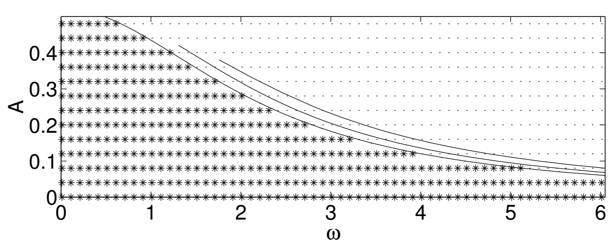

Fig. 4. Partition of the (A, ω) plane. The stars indicate the "no tilt" region. The lower line are the (A, ω) pairs with $t_l = \frac{T}{2} - t_t$. The middle line are the (A, ω) pairs with $t_l = \frac{T}{2}$. And the upper line are the (A, ω) pairs with $t_l = \frac{T}{2} + t_t$.

2) Simulation with collision discontinuities: Converse to the theoretical trajectory planning approach, where an ideal controller was assumed, in simulation we introduce a collision model and a feasible controller for the compensation of the disturbances caused by collision.

As a first approximation to realistic collision models angular momentum conservation is introduced at every landing time. Postulating that the angular momentum before the collision equals the angular momentum after collision and assuming that the velocity of the tilt angle α vanishes immediately after the collision leads to the transition law

$$\begin{pmatrix} \alpha^+ \\ \beta^+ \\ \dot{\alpha}^+ \\ \dot{\beta}^+ \end{pmatrix} = \boldsymbol{h}(\boldsymbol{\zeta}^-) = \begin{pmatrix} 0 \\ \beta^- \\ 0 \\ \dot{\beta}^- + c(\beta)\dot{\alpha}^- \end{pmatrix}.$$

To compensate for the disturbance of unforeseen discontinuities in the velocity of β a computed torque controller is applied. Therefore the control input $\tau(v)$ is chosen such that the differential equation for β transforms to $\ddot{\beta} = v$, where linear control laws can be applied. For underactuated systems an internal dynamics emerges which is the tilting of the foot round its edges. The internal dynamics is stable but not asymptotically stable. The linear control law is chosen to be a PD-controller

$$v = \ddot{\beta}^d + K_D(\dot{\beta}^d - \dot{\beta}) + K_P(\beta^d - \beta), \qquad (1)$$

where $\dot{\beta}^d$ and $\ddot{\beta}^d$ are the derivatives of the desired trajectory for β and K_P and K_D are control constants, here we have chosen $K_P = 100$ and $K_D = 14.14$.

The precomputed periodic trajectories, characterized by the parameter pair (A, ω), are now applied in simulation. From Fig. 5 it can be concluded, that only trajectories with small maximum tilting angle are applicable in realistic conditions, although the precalculation of many more trajectories is possible. The disturbance caused by the landing impact is to high to get compensated before the next discrete state change, which causes unstable hybrid behavior.

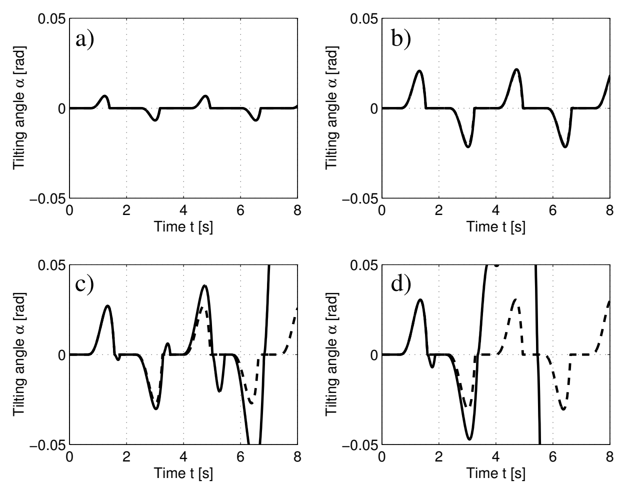

Fig. 5. Simulation Results applying the PD-control law (1) with $K_P = 100$ and $K_D = 14.14$ for different precalculated trajectories. The desired tilting angle α ($\alpha > 0$ for tilting left, $\alpha < 0$ for tilting right) is plotted dashed against the tilting angle in simulation, which is plotted solid. For increasing maximum tilt angle the collision is more and more destabilizing. Finally trajectories are not applicable any more. a) $A = 0.33$, $t_l = \frac{T}{2} - \frac{t_t}{2}$ ($\omega = 1.778189$). b) $A = 0.33$, $t_l = \frac{T}{2} - \frac{t_t}{4}$ ($\omega = 1.844357$). c) $A = 0.33$, $t_l = \frac{T}{2} - \frac{t_t}{6}$ ($\omega = 1.866191$). d) $A = 0.33$, $t_l = \frac{T}{2} - \frac{t_t}{8}$ ($\omega = 1.876994$).

B. Trajectories with focus on the tilt angle

1) Boundary value problem for the trajectories: From the first approach we learned that the collision model as well as the control strategy have to be accounted for in the trajectory design to avoid unwanted destabilization. The key idea here is to utilize the collision impact for the next tilting instead of avoiding collision impacts by control.

We apply the same collision model and the same computed torque PD-controller as in the section before. The function of the controller is to constantly maintain the actuated arm upstanding, that means $\beta = \pi/2$ and $\dot{\beta} = 0$, whereas the assumption of conservation of angular momentum at collision time leads to disturbances in $\dot{\beta}$. A tilted initial configuration $\alpha = \alpha_0$ can be found such that the hybrid tilting orbit, that is the consequence of the initial deflection, is periodic. The situation here is comparable with a rigid block that is put on one of its edges and wobbles to the other edges and back until the energy has dissipated. In our case an internal energy source is provided by the actuated link. In Hogan [10] an external energy source is assumed for the rigid block to approximate its reaction on earthquakes.

For every choice of the control parameters K_P and K_D a resulting periodic orbit is the solution of a boundary value problem. As initial conditions we choose

$$\begin{aligned} \alpha(0) &= 0 \\ \dot{\alpha}(0) &= 0 \\ \beta(0) &= \frac{\pi}{2} + \beta_0 \\ \dot{\beta}(0) &= \dot{\beta}_0. \end{aligned}$$

Here we assume to be at the beginning of tilting left. To obtain periodicity we demand that for an arbitrary time t_{end} a mirrored condition to the initial condition holds. That is

$$\begin{aligned} \alpha(t_{\text{end}}) &= 0 \\ \beta(t_{\text{end}}) &= \frac{\pi}{2} - \beta_0 \\ \dot{\beta}^+(t_{\text{end}}) = \boldsymbol{g}(\boldsymbol{x}^-(t_{\text{end}})) &= -\dot{\beta}_0 \end{aligned}$$

Here we assume to be at the beginning of tilting right. If a solution of the boundary value problem is found tilting right will mirror tilting left and periodicity emerges. The parameters, that are varied to solve the boundary value problem are β_0, $\dot{\beta}_0$ and t_{end}, i.e. three free parameters to meet three boundary conditions.

More demonstrative is the characterization of the orbits by their maximal tilt angle α in a tilting cycle. The maximal tilt angle is obtained by numerical integration, with the initial conditions that are the solution of the boundary value problem. In Fig. 6 some solutions of the boundary value problem are presented.

2) Simulation and chaotic behavior: Not yet proven but concluded from the observation of simulation results is that the periodic orbits obtained from the boundary value problem are not stable. Only very small deviations in the initial values lead to increasing maximal tilt angles or decreasing maximal tilt angles of the periodic orbit.

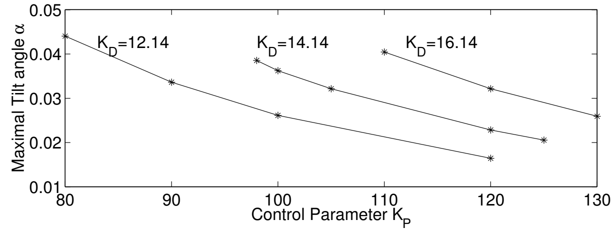

Fig. 6. Solution of the boundary value problem. For several combinations of K_P and K_D the maximal tilt angle α, characterizing the unstable orbit, is calculated.

To stabilize on the hybrid unstable periodic orbit an additional control strategy based on energy conservation is employed.

The average of the total energy of the unstable orbit is

$$E^{av} = \lim_{t^* \to \infty} \frac{1}{t^*} \int_0^{t^*} E(t)\, \mathrm{d}t\,,$$

where $E(t)$ is the total energy which is the sum of potential and kinetic energy for a time t. Similar to the approach from Bühler *et al.* [7] we calculate the energy deviation $\Delta E(t) = E^{av} - E(t)$ of the actual energy $E(t)$ from the average energy E^{av}. The control parameter K_P of (1) is then modified as follows

$$K_P = K_{P0} + k\Delta E$$

to feed energy by high-gain control in situations where too much energy was lost. Simulation results with $K_{P0} = 100$ and $k = 500$ are presented in Fig. 7.

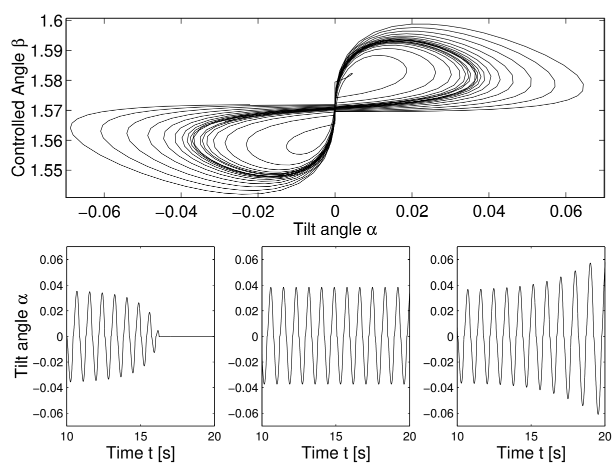

Fig. 7. Stabilization of an unstable periodic orbit by energy dependent adaption of the control parameter K_P. ($K_{P0} = 100$, $k = 500$, $K_D = 14.14$) Top: Phase portrait of the unstable behavior. Either the orbit departs to infinity or the orbit degenerates to "no tilting". The periodic orbit is located in-between. Bottom: Comparison of the tilt angle in an unstable situation with decreasing tilt angle on the left, an unstable situation with increasing tilt angle on the right and controlled on the periodic hybrid orbit in the middle.

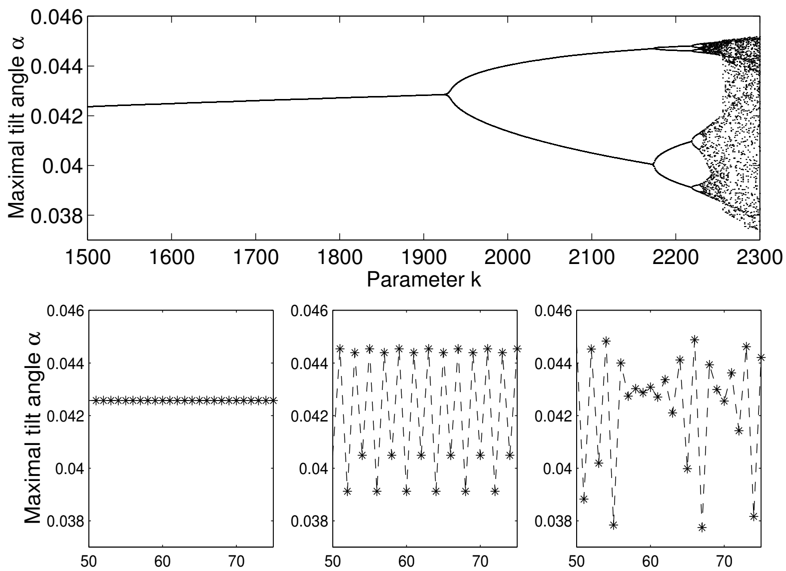

Fig. 8. Top: Bifurcation diagram for variation of the control parameter k. Observed is the maximal tilt angle α in one tilting cycle for increasing control parameter k. For small values of k the tilt angle is constant (Bottom on the left), until, for $k = 1925$ a bifurcation occurs. Here the tilt angle to the left side differs from the tilt angle to the right side (Bottom in the middle). After a few more bifurcations the series of maximal tilt angles is irregular (Bottom on the right). Bottom: Time series of the maximal tilt angle over the tilting cycle n for $k = 1900$, $k = 2210$ and $k = 2300$.

The hybrid orbits show interesting behavior, when k is chosen very high. For $k = 1925$ a first bifurcation occurs, that means the maximal tilt angle of the left tilting motion begins to differ from the maximal tilt angle of the right tilting motion (see Fig. 8). For even higher values of k additional bifurcations occur, resulting in orbits of different period length. Eventually no more regularity is seen in the series of maximal tilt angles and the orbit becomes "chaotic". Similar behavior was shown for the joggling robot by Bühler *et al.* [7].

IV. Conclusions

We have presented two different approaches to compute trajectories for a simple hybrid tilting mechatronic system. For the first approach collisions are negelected in the model, which causes feasibility problems in control: only a subset of the precomputed trajectories is applicable. In the second approach collisions as well as the applied controller is taken into account in trajectory design. The unstable hybrid periodic orbits that we obtain are stabilized by an energy conservation approach. Properties characteristic for chaotic behavior are observed.

The experiences made with the low-dimensional systems are to be applied to higher-dimensional ones. We plan to apply trajectory computation for tilting systems and adequate control strategies to enhance the performance of humanoid robots walking.

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