

# Transparency of the Generalized Scattering Transformation for Haptic Telepresence

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**Abstract.** In this paper we analyze the transparency of the generalized scattering transformation applied to teleoperation systems with constant time delay. Particularly, the human operator, the remote environment, and the robot manipulators are modeled as dissipative instead of just passive systems. By the application of this transformation delay-independent stability is guaranteed. It is shown that transparency can be substantially improved compared to the conventional scattering transformation approach. Simulations and experimental results on a three degree-of-freedom teleoperation system validate the presented approach.

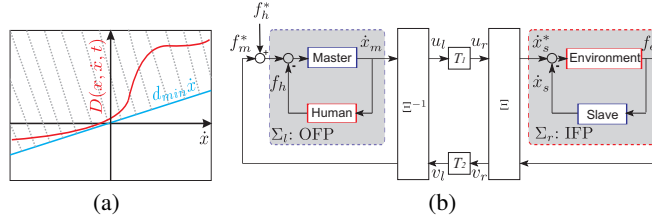
**Keywords:** teleoperation, control, communication unreliabilities, transparency

## 1 Introduction

A haptic telepresence system allows the human to manipulate in remote, inaccessible, dangerous, or scaled environments. From a control point of view, the haptic control loop, where motion and force data are exchanged, is very challenging as it is closed over a communication network, e.g. the Internet. The communication network introduces unreliabilities such as (time-varying) time delay and packet loss, which do not only distort the human haptic perception of the remote environment but potentially destabilize the overall system.

In [7] the authors propose to use the approximate knowledge on damping properties of the human arm, the controlled manipulators, and/or the remote environment. Using the *generalized* scattering transformation [3, 6] which is suitable for dissipative systems delay-independent finite gain  $\mathcal{L}_2$ -stability can be ensured. The contribution of this paper is an extended transparency analysis of the proposed scheme. Simulations and experiments on a three degree-of-freedom (DoF) teleoperation system show that the performance in terms of displayed mechanical properties is improved compared to the standard scattering transformation.

The remainder of the paper is organized as follows: in Section 2 the dissipativity-based modeling of our system and the generalized scattering transformation are presented. The corresponding transparency analysis follows in Section 3 and the experimental evaluation is presented in Section 4.



**Fig. 1.** (a) Nonlinear time-varying damping with lower bound  $d_{min}$  (b) Teleoperation system with time delay and generalized scattering transformation.

## 2 Background

Generally, we assume that the human arm, the environment, and the manipulators can be represented by the second-order dynamics with unknown inertia, damping, and spring parameters in Cartesian space

$$M\ddot{x}(t) + D(x, \dot{x}, t) + Kx(t) = f(t), \quad (1)$$

with  $x \in \mathfrak{R}^n$  the position vector in Cartesian space,  $f \in \mathfrak{R}^n$  the Cartesian external force;  $D(x, \dot{x}, t) \in \mathfrak{R}^n$  is the unknown damping term,  $M \in \mathfrak{R}^{n \times n}$  and  $K \in \mathfrak{R}^{n \times n}$  the unknown diagonal and positive-definite inertia and stiffness matrices, respectively. The components  $D_i(x, \dot{x}, t)$  of the damping term vector  $D(x, \dot{x}, t)$  are assumed to be continuous, and nonlinear functions for which the lower bound  $d_{min} \geq 0$  is known such that  $D_i(x, \dot{x}, t) \geq d_{min}\dot{x}$  holds  $\forall i = 1 \dots n$ ; see Fig. 1(a) for a visualization of the 1-DoF case. The system (1) belongs to the class of input-feedforward-output-feedback passive (IF-OFP) system as it is shown in [7]. Note that passive systems are a special case of IF-OFP systems. Due to space limitations we have to refer the reader to [4] for a more extensive treatment.

In this paper we will exemplarily study a velocity-force architecture with constant time delay as illustrated in Fig. 1(b). We further assume that master and slave manipulator are operated under an admittance control scheme with decoupled dynamics in Cartesian space, i.e. they can be represented by (1). It can be shown, that the controlled manipulators are output-feedback passive (OFP) systems with parameters  $\varepsilon_m = d_{min}^m$  and  $\varepsilon_s = d_{min}^s$ , with subscript 'm', 's', representing the master and slave, respectively. Furthermore, the human arm and the environment are input-feedforward passive (IFP) systems with  $\delta_h = d_{min}^h$  and  $\delta_e = d_{min}^e$ , with subscript 'h' referring to human arm and 'e' to environment. The negative feedback interconnection of the master manipulator and the human arm can be shown to be OFP( $\varepsilon_l$ ) system with  $\varepsilon_l = \varepsilon_m + \delta_h$ . Analogously, the negative feedback interconnection of the environment and the slave manipulator is IFP( $\delta_r$ ) system with  $\delta_r = \min(\delta_e, \varepsilon_s + \delta_e)$  [7].

In order to prevent instability arising from a nonzero time delay between the local and remote side, the *generalized scattering transformation* (GST) is introduced. The GST, a linear input/output transformation, guarantees stability for arbitrary large constant, unknown time delay  $T_1, T_2$ , and it is represented by the matrix  $\Xi$  in Fig. 1(b). Instead of the lefthand output variable  $\dot{x}_m$  the variable  $u_l$  is transmitted over the forward channel, and analogously  $v_r$  instead of  $f_e$  via the backwards channel

$$\begin{bmatrix} u_l \\ v_l \end{bmatrix} = \Xi \begin{bmatrix} \dot{x}_m \\ f_m^* \end{bmatrix}, \begin{bmatrix} u_r \\ v_r \end{bmatrix} = \Xi \begin{bmatrix} \dot{x}_s^* \\ f_e \end{bmatrix} \quad (2)$$

with  $u_r(t) = u_l(t - T_1)$  and  $v_l(t) = v_r(t - T_2)$ . The transformation is parametrized as a rotation  $R$  and a scaling matrix  $B$

$$\Xi = R \cdot B = \begin{bmatrix} \cos \theta I & \sin \theta I \\ -\sin \theta I & \cos \theta I \end{bmatrix} \begin{bmatrix} b_{11} I & 0 \\ 0 & b_{22} I \end{bmatrix}, \Xi = \begin{bmatrix} \xi_{11} & \xi_{12} \\ \xi_{21} & \xi_{22} \end{bmatrix} \quad (3)$$

where  $I$  represents the  $n \times n$  unity matrix,  $\det B \neq 0$  and  $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ .

**Proposition 1.** [3] Assume a system consisting of the networked negative feedback interconnection of an OFP( $\epsilon_l$ ) and an IFP( $\delta_r$ ) system with  $\epsilon_l, \delta_r > 0$ , the bidirectional communication channel with constant time delay, and the input-output transformation (3). The overall system is finite gain delay-independently  $\mathcal{L}_2$ -stable if and only if for each  $B$  the rotation matrix parameter  $\theta \in [\theta_l, \theta_r]$ . Here  $\theta_l$  and  $\theta_r$  are one of the two solutions of

$$\cot 2\theta_i = \frac{\epsilon_{B_i} - \delta_{B_i}}{2\eta_{B_i}}, \quad i \in \{l, r\} \quad (4)$$

which simultaneously satisfy  $a(\theta_i) = 2\eta_{B_i} \sin \theta_i \cos \theta_i - \delta_{B_i} \cos^2 \theta_i - \epsilon_{B_i} \sin^2 \theta_i \geq 0$ ,  $\epsilon_{B_i}$  and  $\delta_{B_i}$  are given by the matrix  $P_{B_i}$

$$P_{B_i} = \begin{bmatrix} -\delta_{B_i} I & \frac{1}{2} I \\ \frac{1}{2} I & -\epsilon_{B_i} I \end{bmatrix} = B^{-T} P_i B^{-1} = \begin{bmatrix} -\delta_i \frac{1}{b_{11}^2} I & \frac{1}{2b_{11}b_{22}} I \\ \frac{1}{2b_{11}b_{22}} I & -\epsilon_i \frac{1}{b_{22}^2} I \end{bmatrix}, \quad i \in \{l, r\}. \quad (5)$$

Hence, instead of choosing  $\theta = 45^\circ$  and  $b_{11} = \sqrt{b}$ ,  $b_{22} = \frac{1}{\sqrt{b}}$ , as for standard scattering transformation [1], here  $\theta$  can be chosen out of an interval.

### 3 Transparency Analysis

A teleoperation system is called transparent if the human user feels like directly interacting with the remote environment. In the following we will analyze the degree of transparency based on mechanical impedances (in contrast to position/force errors). Modeling the real environment as a mechanical linear time-invariant impedance  $Z_e(s)$  and the impedance displayed to the human  $Z_h(s)$  transparency is achieved if [5]

$$Z_h(s) = Z_e(s),$$

where  $s$  is a complex variable, representing the Laplace domain; it will be omitted when not needed. In practice, perfectly transparent teleoperation is difficult to achieve. The interesting question is the possible degree of transparency. The displayed impedance can be computed, here ignoring controller dynamics and robot compliance, based on the environment impedance and the generalized scattering transformation according to

$$Z_h = \frac{\xi_{21} - \xi_{11} R e^{-sT}}{-\xi_{22} + \xi_{12} R e^{-sT}}, \quad R = \frac{\xi_{21} + \xi_{22} Z_e}{\xi_{11} + \xi_{12} Z_e} \quad (6)$$

where  $T = T_1 + T_2$  the round trip time delay. In order to analyze and compare different impedances a Padé approximation is used for the time-delay, which is valid for low

frequencies, i.e.  $e^{-sT} \approx (1 - \frac{T}{2}s) \cdot (1 + \frac{T}{2}s)^{-1}$  for  $\omega < \frac{1}{3T}$ . The teleoperator dynamics are ignored for simplicity. The transformation angle is limited to  $\theta \in [0, \frac{\pi}{2}]$  to simplify the following calculations and the requirement on stable transfer functions.

The displayed impedance in case of an spring-damper environment  $Z_e = \frac{k_e}{s} + b_e$  is approximated for low frequencies by (6) as

$$Z_h^{LF} |_{Z_e = \frac{k_e}{s} + b_e} = \frac{\gamma \frac{k_e}{s} + b_h + m_h s}{1 + \gamma \left( \frac{T}{2} (\cos^2 \theta - \sin^2 \theta) + \frac{b_{22}}{b_{11}} T b_e \sin \theta \cos \theta \right) s} \quad (7)$$

with  $\gamma = \frac{1}{1 + \frac{b_{22}}{b_{11}} T k_e \sin \theta \cos \theta}$ ,  $b_h = \gamma (b_e + \frac{T}{2} k_e (\sin^2 \theta - \cos^2 \theta))$ ,

$m_h = \gamma (\frac{b_{11}}{b_{22}} T \sin \theta \cos \theta + \frac{T}{2} b_e (\sin^2 \theta - \cos^2 \theta))$  and  $\frac{b_{22}}{b_{11}} > \frac{(\sin^2 \theta - \cos^2 \theta)}{2 b_e \sin \theta \cos \theta}$ ; the latter resulting from the requirement on stable transfer functions in the approximation. The denominator of (7) can be shown to be a low pass filter that can be ignored in the lower frequencies. The resulting displayed impedance is then

$$Z_h^{LF} |_{Z_e = \frac{k_e}{s} + b_e} \approx \gamma \frac{k_e}{s} + b_h + m_h s \quad (8)$$

Similarly, it can be shown that for free space motion  $Z_e = 0$  an inertia linearly increasing with the time delay is approximately displayed

$$m_h^{LF} \approx \frac{b_{11}}{b_{22}} T \sin \theta \cos \theta \quad (9)$$

with the computation straightforward from (6).

We observe in (8) and (9) that the scaling matrix alters the performance of the system in a similar way the characteristic impedance of the standard scattering transformation configures the performance, cf. [2]. A small parameter  $\frac{b_{11}}{b_{22}}$  will avoid large inertia in free space movement whereas a large one is required to display high stiffness; it can be chosen freely based on application.

*Numerical Evaluation:* The proposed approach is analyzed in simulation for a velocity - force admittance control scheme consisting of a linear time-invariant spring-damper environment  $Z_e = \frac{500}{s} + 20$  and negligible slave dynamics, i.e.  $\delta_r = \delta_e = 20$ . The lefthandside subsystem is assumed to be OFP( $\varepsilon_l$ ) with  $\varepsilon_l = 10$  resulting from either the human or master dynamics' minimum damping. For comparison reasons a characteristic impedance  $b = 1$  is chosen for the standard scattering transformation and the generalized scattering transformation is tuned with scaling components,  $b_{11} = \sqrt{b} = 1$  and  $b_{22} = \frac{2b_{11} \sin \theta \cos \theta}{b}$  such that in free space motion both methods display same inertia, i.e. have the same free space performance. The resulting system is delay-independently stable for all  $\theta \in [3^\circ, 85^\circ]$ . The time delay is set to  $T_1 = T_2 = 50$  ms. The Bode plots of the environment impedance and the displayed impedance are depicted in Fig. 2(a) for free space motion and in Fig. 2(b) for the stiff environment. It is observed that the displayed impedance is closer to the environment impedance for the generalized scattering transformation. Particularly for the case  $\theta = 30^\circ$ ,  $\theta = 60^\circ$  a stiffness of 48.2 N/m is displayed, whereas for  $\theta = 80^\circ$ ,  $10^\circ$  a stiffness of 204 N/m is displayed. This by far closer to the real environment stiffness than with the standard scattering transformation, where a stiffness of 36.9 N/m is displayed, see Fig. 2(b).

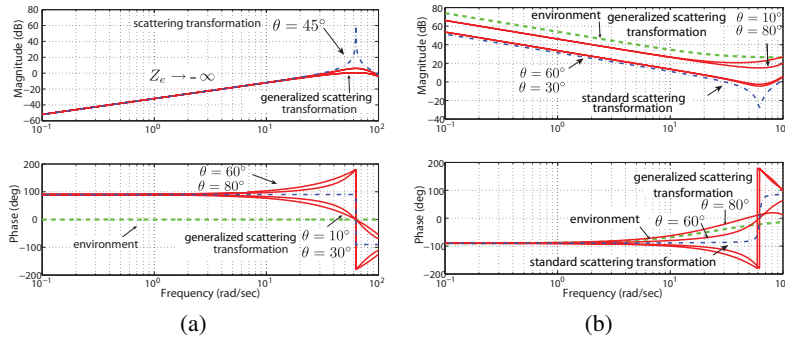


Fig. 2. Displayed impedance comparison in (a) free space (b) contact with the environment

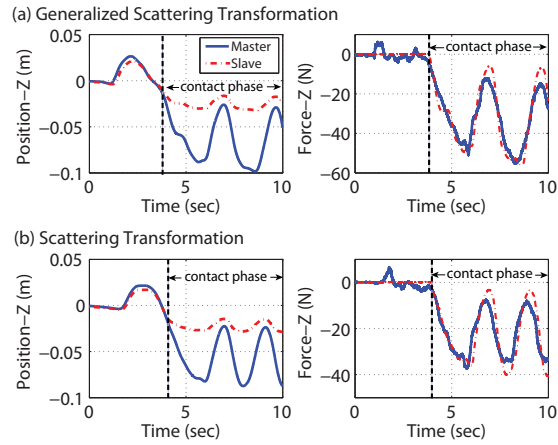


Fig. 3. Experimental setup

## 4 Experiment

The experimental testbed consists of a real teleoperation system with two 4-DoF manipulators. Of the three translational and one rotational DoF only the translational are used resulting in a 3-DoF setup. A 6 DoF force/torque-sensors (JR3) is mounted at the tip of the manipulators to measure interaction forces with the human operator and the environment. The sampling rate of the haptic signals and the local control loops is 1 kHz. A position-based admittance control scheme is considered with a proportional-derivative joint controller; gravity is compensated. The master displayed 12 kg inertia, whereas the slave is rendered as a virtual tool with 40 kg inertia that could display 5000 N/m stiffness in all three degrees-of-freedom.

The master damping is 30 Ns/m in all three degrees-of-freedom, therefore  $\varepsilon_m = 30$ . On the slave side the virtual tool has 1000 Ns/m damping thus  $\varepsilon_s = 1000$ . A silicon cube is used as environment, having a damping 10 Ns/m thus  $\delta_e = 10$ . The human arm damping is considered unknown, i.e. we just assumed passive behavior as typically done in the literature,  $\delta_h = 0$ . The scaling parameters are chosen such that the free-space motion displayed dynamics are similar in both methods,  $b = 800$ ,  $b_{11} = \sqrt{b}$ ,  $b_{22} = \frac{2b_{11} \sin \theta \cos \theta}{b}$  and result in  $\theta_l = 0.1^\circ$  and  $\theta_r = 46.2^\circ$ . The time delay is 50 ms in both channels. The system is stable throughout the experiment. During the experiment the silicon cube, which has a stiffness 1400 N/m, is haptically explored. The proposed scheme is tested with  $\theta = 17^\circ$ , see Fig. 4 for a position and force tracking illustration. The displayed impedance is identified by a least-squares method. The generalized scattering transformation displays an impedance with  $k_h = 492$  N/m and  $b_h = 12$  Ns/m, obviously improved compared to that of the conventional scattering transformation,



**Fig. 4.** Position and force tracking for (a) the generalized scattering transformation and (b) the standard scattering transformation.

$k_h = 367$  N/m and  $b_h = 68$  Ns/m. This generally results in more realistic contact, however, psychophysical studies have to be conducted in order to finally evaluate that.

## 5 Conclusions

In this work, the transparency of the generalized scattering transformation applied to a teleoperation system with constant time delay is analyzed. Simulation and experimental results on a three degree-of-freedom teleoperation system indicate significant improvement, in terms of displayed mechanical impedances, compared to the conventional scattering transformation. In our future work psychophysical studies will aim to look deeper in the transparency results, analyzed in this paper.

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