A Dynamic Model and System-Theoretic Analysis of Affect based on a Piecewise Linear System

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***Abstract*— This work proposes a Piecewise Linear (PL) system to model transitions of affect. Parameters of the model are identified based on a psychological experiment. The PL system describes affective reactions of humans to an external affective stimulus depending on the previous affective state. Results of the statistical analysis support that the previous affective state influences significantly the current affective state. Evaluation of the model shows that it is suitable as mathematical representation for the development of affect over time under the influence of an external stimulus. A following system-theoretic analysis of the model reveals that the PL system shows complex dynamic characteristics. It suggests that internal affective fluctuations exist.**

I. INTRODUCTION

In the last decades, affective computing has received increasing interest in human-computer- (HCI) and human-robot-interaction (HRI). It is expected that integration of emotions enhances HCI [1], [2]. This task implies the following research areas: Recognition of emotions, modeling of emotions as well as emotional interaction, and expression of emotions on a virtual avatar or a robotic head. Emotion recognition in speech, facial expressions and physiology is studied in detail in [3], [4]. Various models exists for classification of emotions in psychology. Most common are the categorical, dimensional and cognitive approach. However, these models concentrate on the description of the current emotional state. Temporal behavior of emotions is largly neglected. A dynamic model of emotions could serve as observer for automatic emotion recognition systems, as tool to optimize affective reactions of a robot and to further investigate the nature of emotions.

Lazarus defines emotion as the combination of physiological disturbance, action tendencies, which are not necessarily acted out, and affect, whereas affect is the subjective experience during an emotion [5]. In accordance with this definition, this study analyzes the temporal development of affect from a system-theoretic point of view. For this purpose, a psychological experiment has been carried out, in which affect has been induced sequentially using pictures of the International Affective Picture System.

Main research questions are:

1) Does the previous affective state in comparison to an external stimulus influence the current affective state?
2) How accurate does the PL system describe the experimental data?
3) What affective behavior do simulations of the PL Sytem predict?

Statistical analysis supports that the previous affective state has a significant influence on the current affective state. Based on experimental data, the parameters of the PL system are identified. Simulations of affective behavior using the PL system indicate that complex dynamics underlie the temporal development of affect. Stability in each region of the PL system depends on the external stimulus.

The paper is structured as follows. Related work is summarized in Section 2. Section 3 describes the psychological experiment which has been conducted to gather experimental data for transition of affect. A statistical analysis investigates the influence of the previous affective state and personality. In Section 4, the parameters of the PL system are identified and the model is evaluated. A system-theoretic analysis regarding stability, observability and reachability is applied to the PL system in Section 5. Main results are discussed in Section 6 and a conclusion is given in Section 7.

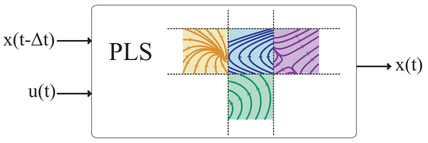

Fig. 1. A Piecewise Linear System is proposed to model the transition of affective states $x(t)$ under the influence of a stimulus $u(t)$.

II. RELATED WORK

Related work can be divided in two groups. On one hand dynamic models are developed, which are used to generate artificial emotions, and on the other hand dynamic models are designed, which are used to investigate and describe human emotions.

The former models have in common that parameters of the model are set heuristically. It is checked in an experiment, if the reaction of the virtual avatar or robot matches expected behavior. Several approaches are taken to generate artificial emotions. Peng et al. propose a Hidden Markov Model and verify its performance in simulations [6]. State equations are considered in [7] to integrate emotions in a service robot. Emotions as part of personality are integrated in [8].

Blewitt et al. present a Millenson-based approach to model emotions in agents based on a fuzzy logic system [9]. Cattinelli et al. consider an interaction model between robot and user based on a probabilistic finite state automaton [10].

To date, understanding the dynamics of human emotions based on a system-theoretic model as well as the relation between system states of the model and real human emotions is widely unconsidered.

III. PSYCHOLOGICAL EXPERIMENT

This study concentrates on a dynamic model and system-theoretic analysis of temporal characteristics of affect. For this purpose, a psychological experiment has been conducted to measure transitions of affective states. Emotions were induced by presenting pictures of the International Affective Picture System (IAPS) to participants [11]. Participants rated their affective states using the Self-Assessment Manikin (SAM) questionnaire [12]. A total of 50 participants took part in the experiment (age: 25.32 ± 2.85, 27 female, 23 male).

The following assumptions regarding psychological aspects are made. It is supposed that the alternating pictures of the IAPS are the primary affective stimulus in the experiment. Furthermore, it is assumed that affect fulfills the Markov property. This means that only the affective state at the previous time step influences the current affective state. Dependencies which go back further in history are neglected. As various classifications for emotions exist in psychology, an analysis of affect is limited to its definition. A categorical as well as dimensional classification of affect is chosen within this study and combined. This covers two out of three main trends.

A. *Description of Affect*

Various theories exist to classify emotions and affect. This study uses a combination of the categorical [13], [14] and the dimensional approach [15]. An affective state is described as a point in the affective state space, which is spanned by the axes pleasure, arousal and dominance [15]. The SAM questionnaire uses this definition. As participants had difficulties to rate the axis dominance using only vision for emotion induction, the following analysis is restricted to the dimensions pleasure and arousal. The basic emotions are anger, disgust, fear, joy, sadness, and surprise [13]. They are derived from facial expressions common among all cultures. In contrast to basic emotions, primary emotions can be blended, so that each emotion consists to a certain part of the primaries acceptance, anger, anticipation, disgust, joy, fear, sadness and surprise [14].

In this work, we combine the dimensions arousal and pleasure with the discrete emotions sad, joyous, neutral and quiet, which are partly derived from the basic emotions, see Fig. 2. The mapping is based on work of Mikels et al. [16] and Morris et al. [17], [18]. Each discrete region contains 16 pictures used for emotion induction.

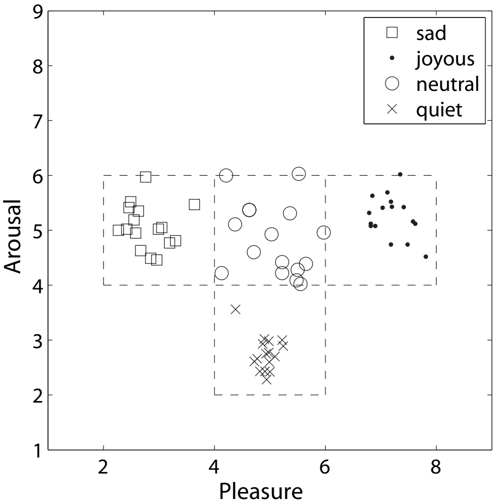

Fig. 2. The four discrete emotional states quiet, sad, joyous and neutral are arranged in a two-dimensional arousal-pleasure subspace of the PAD model. The markers display the affective states of the IAPS pictures used for induction of affect. The dimensions pleasure and arousal range continuously between 1 and 9 according to the SAM questionnaire.

The experiment investigates transitions from one affective state to another under the influence of emotion induction. The transition between each discrete state was repeated twice. Also, repetition of emotion induction within the same region was repeated. Further analysis can be based on the dimensional description of affect, which allows a more detailed analysis, or on discrete labels sad, joyous, neutral and quiet, which are easier generalizable.

B. *Description of the Experiment*

The presentation which includes the IAPS pictures was shown with a video projector. The slide transitions were determined by a timer. Each picture was shown 5 seconds. The participants had 15 seconds time to fill out the SAM

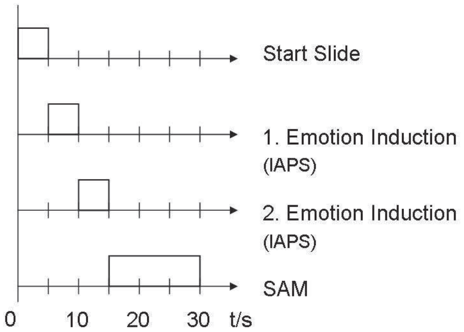

Fig. 3. This graph shows a single cycle to investigate the transition from one affective state to another. First a start slide with the number of the test cycle is shown. After 5 seconds the first IAPS picture is presented. The second emotion induction follows 5 seconds later and the picture is also shown for 5 seconds. Then the SAM questionnaire is filled out. This cycle is repeated 32 times.

questionnaire, which contains a nine-point Likert scale for each dimension pleasure, arousal and dominance. A maximum of 8 persons completed the experiment at the same time. The handout for the participants contained a how-to-do manual for SAM, a mood questionnaire (EWL, in German), SAM questionnaire, and an Eysenck personality questionnaire (EPI, in German). In total, 64 pictures were chosen from the IAPS and each picture was shown only once.

It is supposed that ratings of our participants differ from ratings listed in the IAPS manual for each emotion induction due to cultural differences. To estimate the difference, one picture of each region sad, joyous, neutral and quiet was shown to all participants and rated immediately. Mean and standard deviation for pleasure differ hardly from values in the IAPS manual. However, ratings for arousal were 0.93 lower in average. This shift is taken into account for the expected values of the first emotion induction, which are listed in the IAPS manual. Participants' ratings after the second emotion induction are analyzed statistically below.

C. Statistical Analysis

Statistical analysis[1] investigates if the previous affective state has a significant effect on the current and if personality or mood of the subjects influences the ratings significantly. Depending on the category of the stimulus mean values for arousal and pleasure vary, see Table I and II. It can be noted that induced affective states differ stronger for different second pictures as for different first pictures.

In a first approach, pleasure and arousal are considered as a bivariate observation. Since an analysis of variance approach is considered, we have to perform a multivariate analysis of variance (MANOVA) procedure. The advantage of this approach, as e.g. noted in [20], is that there are no assumptions about the form of the covariance matrix of the repeated measures. Because of that MANOVA is also a valid alternative to analyze univariate repeated measures. The category of the first and second emotion induction are used as within subject factors. The covariate gender is included as between subjects factor in the MANOVA model. Pillai test statistics indicate that besides a significant effect of the second emotion induction ($F(3,46) = 29.45, p = 8,86 \cdot 10^{-11}$), the previous emotion induction significantly influences the current affective state ($F(3,46) = 3.08, p = 0.037$). Also, a significant contribution of the interaction between first and second emotion induction is observed ($F(9,40) = 4.56, p = 0.00035$). There seems to be no main effect of the between-subject factor gender ($F(1,48) = 0.16, p = 0.69$). Hence the data will not be grouped with respect to gender in further analysis.

In [21], it was shown that there can be substantial variation in the within-person pleasure-arousal association. But on average they seem to be independent. Therefore we will analyze the influence of further personality characteristics on pleasure and arousal separately. For both of them, we fitted a linear mixed model (LMM) including the first and second emotion induction as fixed effects and random effects for the second emotion induction of each person, see e.g. [19] for fitting LMM in **R**. As further fixed effects the personality characteristics extroversion and neuroticism are considered. Ratings of the participants can vary from 0 to 24 for extroversion and neuroticism. Both are divided in low (rating < 13) and high (rating > 13). Ratings for mood are standardized to a minimum of 0 and maximum of 1. Due to correlation of 0.54 between the positive and negative mood, we just keep the positive mood as potential covariate. Positive mood is low, if measurement is lower than 0.4, otherwise it is high. Positive mood has a significant effect on arousal ($t = -2.17, p = 0.035$), whereas extroversion has a significant effect on pleasure ($t = -2.59, p = 0.013$). Fig. 4 illustrates the differences of the means for pleasure and arousal graphically.

TABLE I

MEAN VALUES FOR PLEASURE DEPENDING ON THE STIMULUS

2.Picture	1.Picture			
	sad	joy	neutral	quiet
sad	2.9	3.7	2.8	3.4
joy	5.1	7.0	5.7	6.5
neutral	4.7	4.4	5.2	5.4
quiet	4.4	5.5	4.9	5.3

TABLE II

MEAN VALUES FOR AROUSAL DEPENDING ON THE STIMULUS

2.Picture	1.Picture			
	sad	joy	neutral	quiet
sad	5.3	4.7	5.2	5.1
joy	3.5	4.3	4.1	3.6
neutral	4.6	4.6	3.9	3.4
quiet	3.8	2.4	3.1	2.7

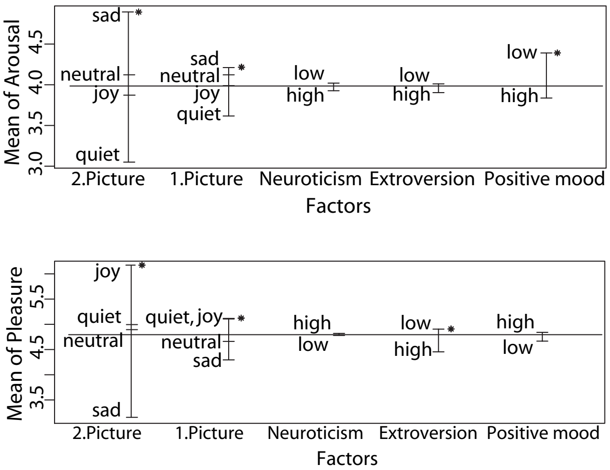

Fig. 4. Means of the participants' ratings are calculated with respect to different factors. Means of arousal differ significantly ($\star$) for the factors 1.picture, 2.picture and positive mood, whereas means of pleasure differ significantly for the factors 1.picture, 2.picture and the personality trait extroversion.

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[1] The statistical analysis was performed in **R** (The **R** Project for Statistical Computing) [19].

Significant influence on affect during watching the second picture has been expected. This only shows that the discrete affective regions sad, joyous, neutral and quiet differ significantly. Significance of the first picture means that the previous discrete state, whether it has been sad, joyous, neutral or quiet, influences the affect during watching the second picture.

IV. PIECEWISE LINEAR SYSTEM

The core contribution of this paper is a dynamic model for transition of affect which calculates the change of affective state $\dot{\mathbf{x}}(t)$ based on the state $\mathbf{x}(t)$ and an input $\mathbf{u}(t)$. The model parameters are estimated based on experimental data of the current and previous affective state and the emotion induction $\mathbf{u}(t)$. It is supposed that transition of affect shows complex behavior, which can be covered by non-linear ordinary differential equations (ODE) in the state space spanned by pleasure and arousal. These complex ODEs are approximated by a set of linear differential equations for each region sad, joyous, neutral and quiet. This Piecewise Linear system (PL) covers the mean affective behavior. As emotion induction has usually a large variance and humans deviate in their reactions, this model describes the most probable affective behavior of an individual in a specific situation.

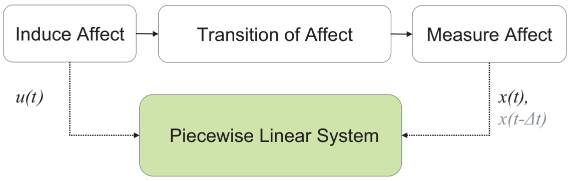

Fig. 5. A dynamic model is designed to model the temporal characteristics of affect. The PL system is estimated based on knowledge of the previous affective state $\mathbf{x}(t-\Delta t)$, the affective stimulus $\mathbf{u}(t)$ and measurement of the current affective state $\mathbf{x}(t)$.

The affective state $\mathbf{x}(t)$ is defined in the pleasure-arousal state space

$$\mathbf{x} = [x_p, x_a]^T, \tag{1}$$

where x_p represents the pleasure rating averaged over all participants and x_a the mean arousal rating. As ratings of the SAM range between 1 and 9, the mean values x_p and x_a range continuously between 1 and 9. It is noted that the state vector does not contain velocity of x_p and x_a, which differs from most technical applications but is common in social or biological context e.g. Lotka-Volterra equations.

Emotion induction is modeled by the input vector $\mathbf{u}$

$$\mathbf{u} = [u_p, u_a]^T, \tag{2}$$

where the components of $\mathbf{u}$ are the expected pleasure value u_p and arousal value u_a for a presented picture, if the previous affective state is neutral. Values are taken from the IAPS manual and corrected by the measured offset.

The output vector $\mathbf{y}$ represents the relation of the affective state $\mathbf{x}$ to the discrete regions sad, joyous, neutral and quiet. Highest affiliation is 1 and lowest 0.

The state space for pleasure and arousal is divided in four regions according to Fig. 2. The behavior in each region q is approximated by the following linear dynamic equations:

$$\begin{aligned} \dot{\mathbf{x}} &= A_q\mathbf{x} + B_q\mathbf{u} + \mathbf{x}_q && (3) \\ \mathbf{y} &= C\mathbf{x} + \mathbf{y}_0 && (4) \\ q &\in \{sad, joyous, neutral, quiet\} \end{aligned}$$

The matrix A_q describes internal fluctuations in human emotions. If no external stimulus is present, affect drifts according to A_q. The impact of the external stimulus $\mathbf{u}$ on $\dot{\mathbf{x}}$ is defined by the matrix B_q. The mapping between the continuous state $\mathbf{x}$ and the discrete affective output $\mathbf{y}$ is realized by the matrix C. This mapping is derived from our definition of the discrete regions q. A constant offset $\mathbf{x}_q$ is introduced so that fixed points can lie arbitrarily in the state space. For the output equation, $\mathbf{y}_0$ is an adequate offset. Sample rate is 0.2 Hz, as pictures have been shown in 5 second intervals.

If the present state $\mathbf{x}$ would not influence the next emotional state and if the input underlies no damping or amplification, the state equations would look as follows:

$$\dot{\mathbf{x}} = \frac{1}{\Delta t}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\mathbf{x} + \frac{1}{\Delta t}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{u} + \frac{1}{\Delta t}\begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5}$$

The matrix B_q is an identity matrices for all q. If mean emotional behavior is completely covered by (5), affective reactions are merely a reaction to affective stimuli and the previous affective state does not influence the current affective state.

If this hypothesis does not hold, the conclusion can be drawn that complex dynamics underlie emotional reactions.

A. Parameter Estimation

The parameters $A_q, B_q, \mathbf{x}_q, C, \mathbf{y}_0$ of the PL system are estimated based on data gathered in the psychological experiment.

Measuring the affective state with the SAM questionnaire is a validated measurement procedure with high reliability in contrast to other approaches like physiological measures, however it does not support continuous recording of affect. To handle this limitation, the derivative $\dot{\mathbf{x}}$ is estimated by:

$$\widehat{\dot{\mathbf{x}}} \cong \frac{\widehat{\mathbf{x}}(\Delta t) - \widehat{\mathbf{x}}(0)}{\Delta t} \quad \text{with} \quad \Delta t = 5s, \tag{6}$$

where $\widehat{\mathbf{x}}(0)$ is the current expected affective state induced by emotion induction with the first picture. Its value is the corrected value from the IAPS manual. The input $\widehat{\mathbf{u}}(\Delta t)$ is the second emotion induction, which interacts with the current affective state $\widehat{\mathbf{x}}(0)$ and leads to a change $\widehat{\dot{\mathbf{x}}}$. The mean $\widehat{\mathbf{x}}(\Delta t)$ of the measured affective state, after the second picture was shown for 5 seconds, is calculated over all participants. This leads to the following equation:

$$\widehat{\mathbf{x}}(\Delta t) = (\Delta t \cdot A_q + I)\widehat{\mathbf{x}}(0) + \Delta t \cdot B_q\widehat{\mathbf{u}}(\Delta t) + \Delta t \cdot \mathbf{x}_q \tag{7}$$

Eq. 7 can be separated for $\widehat{x}_p(\Delta t)$ and $\widehat{x}_a(\Delta t)$, each is an over-determined equation with five unknowns and eight measurements for each region. The parameters A_q, B_q and $\mathbf{x}_q$ are identified by Least Squares optimization.

This results in the following state equations.
State equation for the region joy:

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{pmatrix} -0.08 & -0.13 \\ 0.13 & -0.15 \end{pmatrix}\mathbf{x}+ \\ &+ \begin{pmatrix} 0.11 & -0.07 \\ -0.02 & 0.12 \end{pmatrix}\mathbf{u}+\begin{pmatrix} 0.65 \\ -0.75 \end{pmatrix} \qquad (8)\end{aligned}$$

State equation for the region neutral :

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{pmatrix} 0.14 & -0.06 \\ -0.04 & -0.30 \end{pmatrix}\mathbf{x}+ \\ &+ \begin{pmatrix} 0.16 & -0.07 \\ -0.01 & 0.04 \end{pmatrix}\mathbf{u}+\begin{pmatrix} 0.46 \\ 1.43 \end{pmatrix} \qquad (9)\end{aligned}$$

State equation for the region sad:

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{pmatrix} -0.16 & 0.01 \\ 0.35 & -0.19 \end{pmatrix}\mathbf{x}+ \\ &+ \begin{pmatrix} 0.09 & -0.01 \\ -0.03 & 0.23 \end{pmatrix}\mathbf{u}+\begin{pmatrix} 0.32 \\ -1.11 \end{pmatrix} \qquad (10)\end{aligned}$$

State equation for the region quiet:

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{pmatrix} -0.05 & -0.03 \\ -0.12 & 0.08 \end{pmatrix}\mathbf{x}+ \\ &+ \begin{pmatrix} 0.14 & -0.02 \\ -0.05 & 0.19 \end{pmatrix}\mathbf{u}+\begin{pmatrix} -0.23 \\ 0.45 \end{pmatrix} \qquad (11)\end{aligned}$$

The parameters of A_q vary for each region. This supports that an individual model for each region is required. The elements of B_q are similar. As expected, the elements on the diagonal are larger than the elements on the secondary diagonal so that the component arousal of the stimulus increases arousal and reduces pleasure of the state and vice versa. Eq. (8) to (11) differ from (5) with $\Delta t = 5s$. It can be noted that the previous affective state influences the current affective state.

The output describes the mapping between the continuous states pleasure and arousal to the four discrete states joyous, neutral, sad and quiet. The matrix C and the offset $\mathbf{y}_0$ need to be estimated. Switching surfaces S_1, S_2 and S_3 are defined between the regions joyous, neutral, sad and quiet. Their mathematical definition is as follows:

$$\begin{aligned} S_1 &= \{\mathbf{x}|f_1(\mathbf{x})=0\} \quad , \quad f_1 = [1\,0]\mathbf{x}-4.0 \\ S_2 &= \{\mathbf{x}|f_2(\mathbf{x})=0\} \quad , \quad f_2 = [1\,0]\mathbf{x}-6.0 \qquad (12) \\ S_3 &= \{\mathbf{x}|f_3(\mathbf{x})=0\} \quad , \quad f_3 = [0\,1]\mathbf{x}-3.1 \end{aligned}$$

As participants in our experiment rated in average arousal 0.93 lower compared with values in the IAPS manual, the function f_3 between quiet and neutral is shifted to lower arousal values compared to Fig. 2. The signs of f_1, f_2 and f_3 differ depending on the affective state, e.g. for a specific state in the region for neutral, f_1 and f_3 are positive and f_2 is negative. Table III lists the signs of f_1, f_2 and f_3 in each region.

The first element of the vector calculated by $C\mathbf{x}_k + \mathbf{y}_0$ is associated with the discrete state sad, the second with joy, the third with neutral and the last with quiet. The following equation combines superposition of the functions f_1, f_2 and f_3 and normalization to values between 0 and 1.

$$\mathbf{y} = \begin{pmatrix} -0.0719 & 0.0360 \\ 0.0719 & 0.0360 \\ 0 & 0.0360 \\ 0 & -0.0360 \end{pmatrix}\mathbf{x}+\begin{pmatrix} 0.6763 \\ 0 \\ 0.4101 \\ 0.6252 \end{pmatrix} \qquad (13)$$

Depending on the highest component in $\mathbf{y}$, the continuous state $\mathbf{x}$ is assigned to this region

TABLE III

VALUES OF FUNCTIONS IN AFFECTIVE REGIONS

region	f_1	f_2	f_3
sad	-	-	+
joy	+	+	+
neutral	+	-	+
quiet	+	-	-

TABLE IV

RMS ERROR OF SIMULATION IN COMPARISON TO EXPERIMENTAL DATA

	region			
	sad	joy	neutral	quiet
pleasure	0.3	0.1	0.3	0.2
arousal	0.1	0.3	0.3	0.3

B. Evaluation

The performance of the model is compared to experimental data. The rms error e_j is calculated for each of the 32 transitions. Table IV lists the rms error for each region. Besides rms errors, the means of measured transitions of each region have been compared with means of equivalent simulated transitions. Mean values of simulations show same order as for experimental data, e.g. that pleasure is highest for a sad stimulus, if the previous affective state has been joyous.

V. SYSTEM-THEORETIC ANALYSIS

Stability of dynamic systems is an interesting characteristic from a system-theoretic point of view. According to Lyapunov's indirect method, eigenvalues of the state matrix A_q determine the characteristic of each fixed point, which is independent of the external stimulus $\mathbf{u}$. Calculation of the eigenvalues of A_q shows, that the fixed point of each region is stable, see Tab. V.

TABLE V

EIGENVALUES OF THE STATE MATRIX A_q

region q	1st Eigenvalue	2nd Eigenvalue	fixed point
sad	-0.121	-0.233	stable
joy	$-0.117+0.126i$	$-0.117-0.126i$	stable
neutral	-0.132	-0.317	stable
quiet	-0.003	-0.131	stable

Position of the fixed points within each region shift depending on the input $\mathbf{u}$. Next, exemplar progressions of

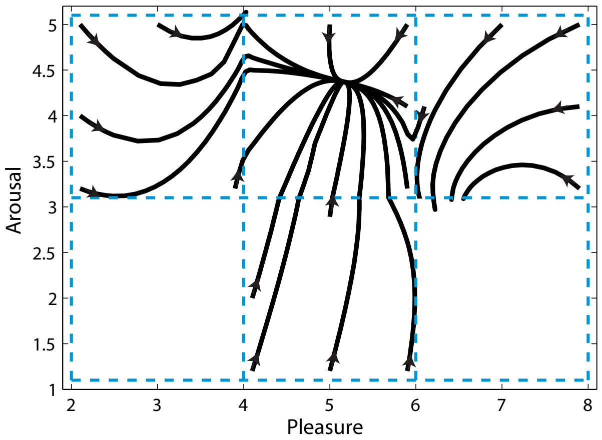

Fig. 6. The fixed point in the neutral region is reached from bordering regions for a neutral stimulus.

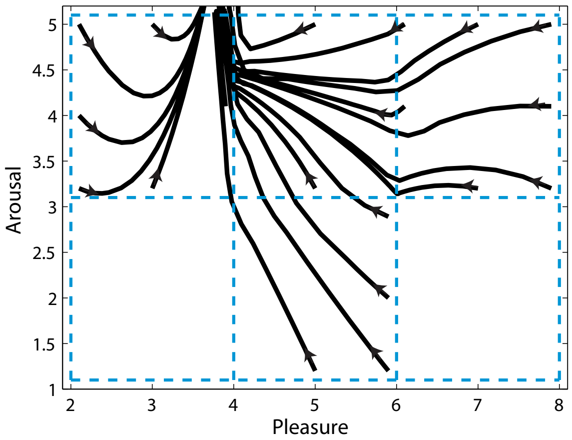

Fig. 7. Repeating a sad stimulus shifts the stable fixed point in the region sad to high arousal and low pleasure. However, the fixed point lies outside the sad region.

the complete system over time are discussed for a constant neutral, sad, quiet and joyous stimulus.

Fig. 6 shows the phase diagram for a constant neutral stimulus $\mathbf{u}_{const} = [5,4]^T$. If the initial affective state is in the regions sad, quiet or neutral, the fixed point is reached. Trajectories with high pleasure and high arousal also reach the neutral fixed point. But not all trajectories of the joyous region stay within the regions of the PL system covered by experimental data.

Fig. 7 to 9 illustrate the phase diagrams for a constant sad stimulus $\mathbf{u}_{const} = [3,4]^T$, joyous stimulus $\mathbf{u}_{const} = [7,4]^T$ and quiet stimulus $\mathbf{u}_{const} = [5,2]^T$. Locations of the fixed points shift for each region depending on the stimulus. Global behavior of the PL system for each stimulus is in accordance with expected behavior. For repetition of a sad stimulus, no matter what the previous state is, pleasure decreases and arousal increases. Repeating a joyous stimulus, increases pleasure in all regions. Repetition of a quiet stimulus leads to a decrease of arousal in the regions sad, quiet and joyous.

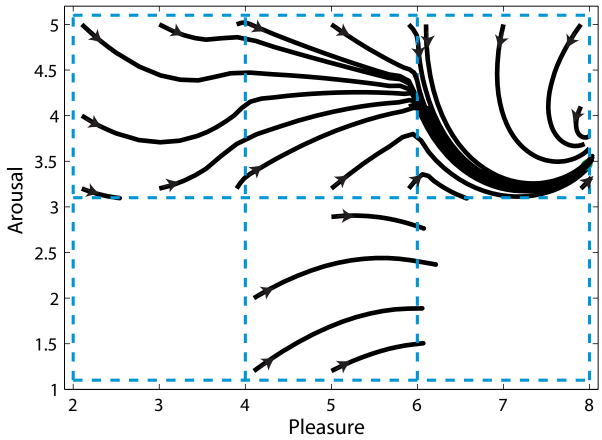

Fig. 8. Repetition of a joyous stimulus increases pleasure for all initial states.

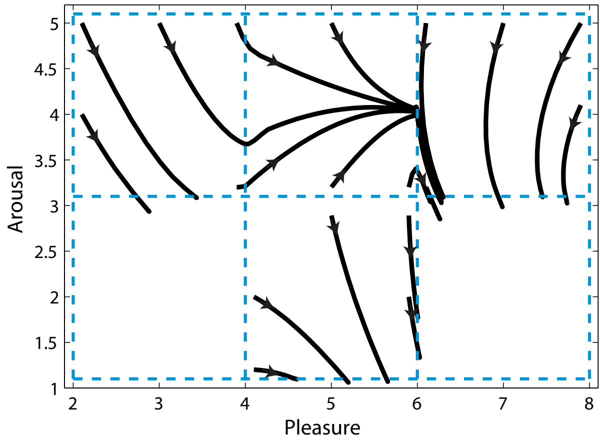

Fig. 9. Repeating a quiet stimulus decreases arousal. It seems in the neutral region as if trajectories end in a stable fixed point. But before the fixed point is reached, the PL system switches to the joyous region.

If the initial state lies within the neutral regions, it seems as if the trajectories reached a stable fixed point. However, the PL system switches to the region joyous, before the fixed point of the neutral region is reached and arousal decreases.

Observability for PL systems can be analyzed analytically. The mapping $\mathbf{x} \mapsto \mathbf{y}(\mathbf{x})$ is injective, so that $\mathbf{x}$ is observable [24]. However, knowledge only of the affective label sad, joyous, neutral and quiet, which corresponds to the highest component in $\mathbf{y}$, is not sufficient to estimate $\mathbf{x}$.

Analysis of reachability is based on simulations of reachable sets. Starting from each point in the four regions with a solution of 0.1, the trajectories for all inputs $\mathbf{u}$ have been calculated. If each region is investigated separately, the four regions are reachable. However, combination of reachable subsystems does not necessarily lead to reachability of the complete PL system. In our case, not all states of the complete PL system can be reached with a constant input $\mathbf{u}$, so that the complete PL system is reachable only for combinations of different inputs $\mathbf{u}$.

VI. DISCUSSION

Evaluation of the PL system shows that the model is capable to cover transitions of affect for the defined regions, so that system-theoretic analysis can be applied and interpreted concerning temporal development of affect. Furthermore, the previous affective state has an important influence on the reaction to an external stimulus. Affective behavior is not merely reaction to external stimuli, also complex internal fluctuations exist.

Results for stability of affective states depending on different external stimuli is in accordance with expected behavior and gives additional insights in the complex structure of affect. Though, a PL system is only an approximation for a non-linear system, it is capable to approximate non-linear characteristics. It can be noted that behavior in each region is different. This corroborates the initial assumption that complex non-linear dynamics underlie internal affective fluctuations.

The model is parameterized based on experimental data gathered by applying different visual stimuli, so that it is limited to a constrained scenario. Still it is shown, that the approach is sufficient to cover complex dynamics of affect. Further limitations of the model are that other influences on affect, such as high inter- as well as intra-individual differences, mood and personality, are neglected so far.

VII. CONCLUSION

A Piecewise Linear system is proposed to model mean affective reactions of humans to external stimuli under consideration of the previous affective state. The model is based on data gathered in a psychological experiment. Statistical analysis shows that the previous affective state influences significantly the current affective state. Performance of the model is evaluated in comparison to experimental data. System-theoretic analysis of the PL model for affect reveals that each region has a stable fixed point, which can be reached depending on the external stimulus. Furthermore, it is indicated that complex dynamics underlie internal affective fluctuations. In principle, the characteristics of the PL system covers transitions of human affect, however the model is limited to a constrained scenario. Applications of the model are as an estimator of the next emotional state under the influence of an external stimulus as well as knowledge of the previous affective state in emotion recognition, as human-like emotion core in an affective avatar or robotic head and as mathematical description to further investigate affect. Future work includes detailed investigation of existence of fixed points for all possible external stimuli, changes of global behavior depending on external stimulus and integration of individual parameters like mood and personality in the model.

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