Switching Control for a Networked Vision-based Control System

Eine schaltende Regelung für ein vernetztes bildbasiertes Regelungssystem

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Summary Advances in communication and computation technology facilitate closed loop control via communication networks. Such networked control systems may include distributed sensors and actuators as well as distributed computational resources. In the field of vision-based control, the possibility of distributed computation is of particular interest due to the large amounts of visual data to be processed. Time-delay caused by data transfer through the network and by processing algorithms impair stability and performance of the controlled closed-loop system. In this article, a novel switching control scheme for networked vision-based control systems is proposed accounting for varying time-delay in order to improve control performance. The closed-loop system is modeled as a continuous system with varying time-delay considering processing and transfer time-delays as well as sampling intervals. Mean exponential stability is proven based on occurrence probabilities of delays. The article concludes with comparative experiments showing a significant improvement of control performance of the proposed approach with respect to a non-switching reference controller. ►►► **Zusammenfassung** Fortschritte in Kommunikations- und Rechentechnik ermöglichen regelungstechnische Anwendungen über Kommunikationsnetze. Derartige vernetzte Regelungssysteme können sowohl verteilte Sensoren und Aktoren als auch verteilte Rechenressourcen beinhalten. Insbesondere im Bereich der bildbasierten Regelungen ist die Möglichkeit verteilter Rechenressourcen aufgrund der hohen anfallenden Datenmengen interessant. Durch den Datenaustausch über das Kommunikationsnetz und Verarbeitungsalgorithmen entstehende Zeitverzögerungen beeinträchtigen jedoch Stabilität und Regelgüte. Dieser Artikel schägt ein neuartiges schaltendes Regelungskonzept für vernetzte bildbasierte Regelungssysteme vor, das variable Zeitverzögerungen berücksichtigt, um die Regelgüte zu verbessern. Die Modellierung des Gesamtsystems unter Berücksichtigung der Rechen- und Übertragungszeitverzögerungen sowie der Abtastintervalle erfolgt über ein kontinuierliches System mit stochastischen Zeitverzögerungen. Die exponentielle Stabilität der Mittelwerte wird basierend auf der Auftrittswahrscheinlichkeit der Zeitverzögerungen sichergestellt. Abschliessende Experimente mit einem bildbasierten Regelungssystem in Form eines kamerageführten Linearaktors validieren den vorgeschlagenen Ansatz. Die experimentellen Ergebnisse zeigen eine signifikante Verbesserung der Performanz durch den vorgeschlagenen Ansatz gegenüber einem Referenzsystem mit nichtschaltendem Regler.

Keywords Stochastic stability, random time-delay, sampled-data systems, networked control systems, visual servoing ►►► **Schlagwörter** Stochastische Stabilität, stochastische Zeitverzögerung, Abtastsysteme, digital vernetzte dynamische Systeme, bildbasierte Regelung

1 Introduction

Due to their affordability and well-developed infrastructure, communication networks are widely used for the signal transmission in control systems. Replacing conventional point-to-point connections with a digital communication network gives a system flexible reconfiguration capability. Furthermore, distributed sensors and actuators can be easily added to this networked control system (NCS) without additional wiring effort. However, with an increasing number of sensors and therefore increasing amount of data, e. g. multiple image streams, requirements for computational resources for estimating relevant variables increase. It might be useful to use distributed networked computational units for performing such information processing tasks [1]. As a result the computation time as well as communication quality between sensors, computational units, and actuators may affect the closed loop behavior of the system see Fig. 1 for a visualization. Distributed computation can efficiently integrate multimedia sensor data of NCSs to increase their control accuracy and flexibility of deployment. Examples are vision-based manipulation [2], coverage control [3], environmental monitoring, and surveillance [4].

Despite of many potential advantages, the networked solution for distributed sensing and computation in control systems introduces several issues that need to be addressed. i) Exchanging sensor data over a communication network results in non-ideal signal transmission. Random packet dropouts and delays may affect the data transmission and thereby the closed loop behavior. ii) There is a computation delay, which can be modeled random due to the existence of conditional branches and resource sharing in distributed computation [5]. iii) Sensor data are sampled not equidistantly in time. This results in aperiodic sampling intervals for the closed loop control system. Time-delay might be a source of instability and deteriorate the control performance [6]. In the past NCS research, mainly network-induced delays have been considered, see e. g. [7; 8]. In this article, the focus is on networked control systems with random computation and communication delay as well as non-equidistant sampling intervals.

Various control approaches for NCS have been proposed in the literature that approximate a random time-delay by its upper bound, i. e. consider the *worst case*. Potentially available stochastic models in terms of probability distributions of the delay are discarded. Typically, this results in conservative controller design for systems with random delay.

Less conservative controller design approaches based on stochastic analysis and random delays are proposed

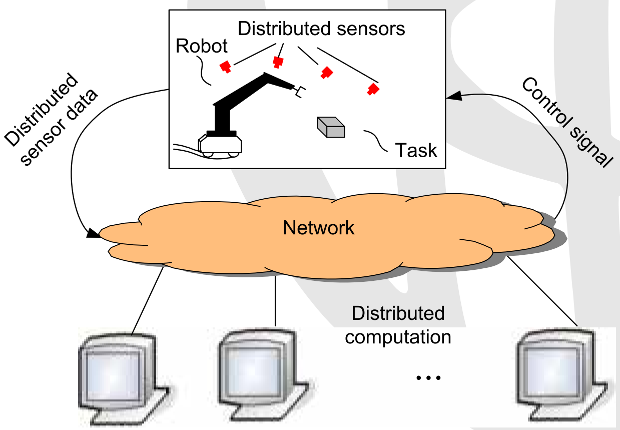

Figure 1 Scheme of networked control systems with distributed sensors and distributed computation.

in [9–14] and [15] for a general overview. In [9], the random delay is considered as an i. i. d. binary random process. The associated stability conditions and controller design algorithms are derived using the statistical properties of delays. A Markov process is used to model the random delays as stochastic process in [10–14]. The resulting closed-loop systems are Markovian jump systems (MJSs) with stability conditions and controller design algorithms being determined either by the infinitesimal generator of the delays in continuous-time modeling or by the transition matrix of the delays in discrete-time modeling. However, these works do not address aperiodic sampling intervals.

For communication time-delay i. i. d. processes or Markov processes are popular models in the communication community [16]. The choice largely depends on the communication protocol and concurrent traffic characteristics. Stochastic models for computation delay are less considered in the literature. In a first approach we will consider the computational delay to be an i. i. d. process. According to [17], the addition of two i. i. d. processes is again an i. i. d. process, whereas the addition of an i. i. d. and a Markov process results in a hidden Markov process. Hence, the sum of transmission and computation delays is either an i. i. d. or a hidden Markov process. In our specific setting we will consider a sensor which is read whenever the processing of the previously sampled data is finished. Such settings are common for example in image sensors. In consequence, the sampling interval is equal to the computational delay and has the same stochastic properties. In this work we consider i. i. d. transmission and computation delay.

Using the representation of aperiodic sampling intervals by a time-varying delay, the compound delay for the closed-loop system comprises transmission and computation delays as well as the delay induced through aperiodic sampling. The random compound delay is categorized into n intervals, whose probabilistic occurrence is described by a set of indicator functions. For good control performance, a delay-dependent switching controller is proposed. The resulting closed-loop system becomes a randomly switched time delay system. A delay-dependent stability condition is derived by a Lyapunov-Krasovskii functional and the associated delay-dependent switching controller design algorithm is presented in terms of LMI. Both, stability condition and controller design algorithm are determined by the occurrence probabilities of delays. The proposed approach is experimentally validated in a networked visual servoing system. The experimental results demonstrate significant performance improvements by the proposed approach over the standard non-switching control method.

The remainder of the paper is organized as follows: The problem statement of an NCS with aperiodic sampling intervals as well as random transmission and computation delays is given in Sect. 2. In Sect. 3, a randomly switched time delay system is introduced by using the input-delay approach. An exponential mean-square stability condition is derived in Sect. 4 and an LMI controller design algorithm is established in Sect. 5. In Sect. 6, the experimental validation and performance comparison are discussed.

Notation. In this paper $\lambda_{\max}(M)$ and $\lambda_{\min}(M)$ denote the maximal and the minimal eigenvalues of a matrix M, whereas M^T and $||M||$ denote the transpose operator and induced Euclidean norm of matrix (or vector) M, respectively. The symbol $*$ denotes the transpose of the blocks outside the main diagonal block in symmetric matrices. $\mathbb{E}$ stands for mathematical expectation and **Pr** for probability. $\mathbb{R}^n$ denotes the subset of n-dimensional real vectors and $\mathbb{N}$ is the subset of natural numbers.

2 Problem Formulation

In this section we will derive the model of a closed loop system with random time-delays and aperiodic sampling intervals. Consider a linear continuous-time plant

$$\dot{x}(t) = Ax(t) + Bu(t)\,, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^q$ is the control input; A, B are constant matrices of appropriate dimensions and (A, B) is controllable.

Traditionally, a sampled-data system with zero-order hold (ZOH) and discrete-time controller has the control input expressed by

$$u(t) = Kx(t_k)\,, \quad t \in [t_k, t_{k+1})\,, \quad \forall k \in \mathbb{N} \tag{2}$$

where t_k denotes the sampling instant. Under the control law (2), the closed-loop system is derived as

$$\dot{x}(t) = Ax(t) + BKx(t_k)\,. \tag{3}$$

For feedback systems with periodic sampling and ideal data transmission channels, the closed-loop system in (3) is equidistantly updated by

$$h = t_{k+1} - t_k\,.$$

In this case, the lifting technique [18] is applied to derive stability certificates and appropriate control algorithms.

For NCSs with aperiodic sampling intervals, random computation and transmission delays, the sampled data $x(t_k)$ arrives at the controller in a non-deterministic pattern. Throughout the article, we assume that packets do not overtake each other, i. e. packets arrive at the receiver according to their sending order. The timing diagram of the sampled measurement is shown in Fig. 2. The sampling of $x(t_k)$ is triggered when the processing of $x(t_{k-1})$ is finished. For further analysis we assume that the computation delays and the sampling intervals take values in a finite set. This is a reasonable assumption as computational units are typically clocked in fixed time slices. Therefore, the sampling interval at the sensor side is

$$t_{k+1} - t_k = \tau_k^c\,, \quad \tau_k^c \in \mathrm{T}^c = \{\mathrm{T}^{c1}, \mathrm{T}^{c2}, \ldots, \mathrm{T}^{cp}\}\,, \quad p \in \mathbb{N}$$

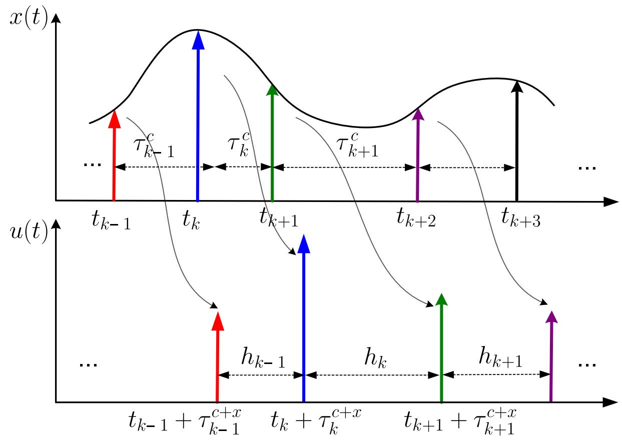

Figure 2 Timing diagram of NCSs with random computation delay τ_k^c, transmission delay τ_k^x and aperiodic sampling. h_k is the holding delay between two consecutive updates at the controller.

where $\tau_k^c \geq 0$ represents the computation delay for the k-th sample, and T^{ci} with $i = 1, \ldots, p$ represent the possible realizations of the computation delay. The computation delay is assumed to be i. i. d. which turns out to be a valid approximation as confirmed in experiments. As shown in Fig. 3, $x(t_k)$ arrives at the controller with the delay

$$\tau_k^{c+x} = \tau_k^c + \tau_k^x\,, \quad \tau_k^x \in \mathrm{T}^x = \{\mathrm{T}^{x1}, \mathrm{T}^{x2}, \ldots, \mathrm{T}^{xq}\}\,, \quad q \in \mathbb{N}$$

where $\tau_k^x \geq 0$ represents the transmission delay for the k-th sample, and T^{xi} with $i = 1, \ldots, q$ are the possible values of the transmission delay. Also the transmission delay is assumed to be a random i. i. d. process. As a result, the closed-loop system in (3) becomes

$$\dot{x}(t) = Ax(t) + BKx(t_k)\,, \quad t \in [t_k + \tau_k^{c+x}, t_{k+1} + \tau_{k+1}^{c+x})\,. \tag{4}$$

The closed-loop system in (4) is updated in non-deterministic patterns

$$\begin{aligned} h_k &= t_{k+1} - t_k + \tau_{k+1}^{c+x} - \tau_k^{c+x} \\ &= \tau_{k+1}^{c+x} - \tau_k^x \end{aligned} \tag{5}$$

depending on delays τ_{k+1}^{c+x}, τ_k^x. Due to the randomness of τ_{k+1}^{c+x} and τ_k^x, the lifting technique is no longer straightforwardly applicable.

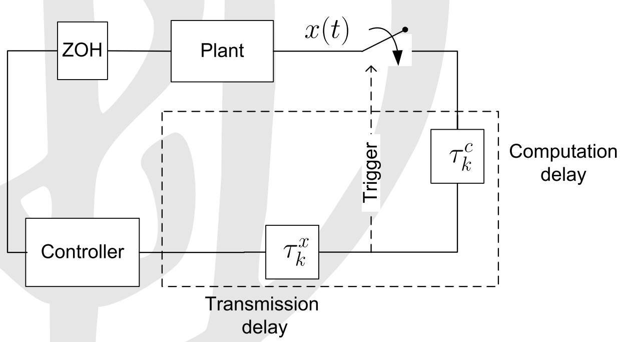

Figure 3 Illustration of NCS with distributed computation. Sampling of the sensor data is triggered when the processing of the previously sampled data is finished.

The challenge to be addressed in this article is to develop a control algorithm such that the closed-loop system in (4) is mean exponentially stable (MES) according to the following definition.

Definition 1. *A diffusion process $x(t)$ is said to be MES if*

$$\mathbb{E}\left\{||x(t)||^2 \mid x(t_0)\right\} \le b||x(t_0)||^2 e^{-\rho(t-t_0)}, \tag{6}$$

where $b > 0$, $\rho > 0$ are real numbers and $x(t_0)$ is the initial condition.

3 System Reformulation

In this section, the NCS with aperiodic sampling intervals, random computation and transmission delays is reformulated into a time-varying delay system by means of the input-delay approach. A switching control mechanism is proposed which switches depending on the value of the overall time delay.

3.1 Input-Delay Transformation

Reconsider the sampling instant t_k as

$$\begin{aligned} t_k &= t - (t - t_k) \\ &= t - \tau(t), \quad t \in \left[t_k + \tau_k^{c+x}, t_{k+1} + \tau_{k+1}^{c+x}\right). \end{aligned}$$

The time-varying delay $\tau(t)$ is bounded by $\underline{\tau} \le \tau(t) \le \bar{\tau}$, where

$$\begin{aligned} \bar{\tau} &= \max_{k\in\mathbb{N}}\{\tau_k^{c+x} + h_k\} = \max_{k\in\mathbb{N}}\{\tau_{k+1}^{c+x} + \tau_k^c\}, \\ \underline{\tau} &= \min_{k\in\mathbb{N}}\{\tau_k^{c+x}\}. \end{aligned} \tag{7}$$

Due to the assumption of no out-of-order arrivals, the derivative of $\tau(t)$ between two consecutive updates is in consequence $\dot{\tau}(t) = 1$. Substituting $x(t_k) = x(t - \tau(t))$ into the closed-loop system in (4), it results in a continuous-time system with time-varying delay

$$\begin{aligned} &\dot{x}(t) = Ax(t) + BKx(t - \tau(t)), \\ &t \in \left[t_k + \tau_k^{c+x}, t_{k+1} + \tau_{k+1}^{c+x}\right), \end{aligned} \tag{8}$$

with initial condition $x_0 = x(\theta)$, $\theta \in [-\bar{\tau}, 0]$.

Remark 1. The reformulation of a sampled-data system into a continuous-time system with time-varying delay is called input-delay approach. This approach is first introduced in [19; 20] where systems with periodic sampling are considered. A similar approach for systems with aperiodic sampling is addressed in [21], but without considering time-delay. In this article we combine those approaches and consider aperiodic sampling intervals and random time-delays conjointly.

Hence, the sampled-data system is reformulated into a continuous-time system with time-varying delay with input-delay approach.

3.2 Time-Delay Model

In order to improve the control performance, a delay-dependent switching controller, which switches its feedback gain according to the current delay value, is introduced. The control law is defined by

$$u(t) = K_{i(\tau(t))}x(t - \tau(t)), \quad i = 1, \dots, n, \tag{9}$$

where $K_{i(\tau(t))}$ represent the state feedback controllers that switch depending on the size of the time delay $\tau(t)$. Therefore the time varying delay $\tau(t)$ is categorized into n finite intervals

$$\begin{aligned} \tau_1 &= \{\tau | s_0 \le \tau < s_1\}, \\ \tau_2 &= \{\tau | s_1 \le \tau < s_2\}, \\ &\vdots \\ \tau_n &= \{\tau | s_{n-1} \le \tau < s_n\}, \end{aligned} \tag{10}$$

where $s_i > 0$ satisfying $s_i < s_{i+1}$, for $i = 1, \dots, n-1$, and $s_0 = \underline{\tau}$, $s_n = \bar{\tau}$. The closed-loop system in (8) can be rewritten as

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{n} \beta_i BK_i x(t - \tau(t)), \tag{11}$$

where β_i is the indicator function

$$\beta_i = \begin{cases} 1, & s_{i-1} \le \tau < s_i, \quad i = 1, \dots, n \\ 0, & \text{otherwise}, \end{cases} \tag{12}$$

and the dependence of the index i of $\tau(t)$ is omitted for simplicity of notation. For the ease of later stability analysis, the worst-case of each delay interval τ_i is considered [13], i. e. s_i, which results in the closed-loop system

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{n} \beta_i BK_i x(t - s_i), \tag{13}$$

for stability analysis.

In consequence, the statistical properties of the category bounds s_i become important. In the following we will show that the i. i. d. property of the computation delay τ_k^c and the transmission delay τ_k^x is inherited through a special choice of the category bounds s_i, $i \in [1, n-1]$. Observe that $\tau(t)$ is composed of the random computation delay τ_k^c, the transmission delay τ_k^x and the holding delay $\tau^h(t) \in [0, h_k)$ caused by non-deterministic updates h_k at the controller

$$\tau(t) = \tau_k^c + \tau_k^x + \tau^h(t), \quad t \in \left[t_k + \tau_k^{c+x}, t_{k+1} + \tau_{k+1}^{c+x}\right).$$

Further observe, that $\tau(t)$ forms a sawtooth function with slope $\dot{\tau}(t) = 1$ between two consecutive updates, see Fig. 4.

Remember that random delays τ_k^c, τ_k^x are modeled by i. i. d. random processes and define

$$\begin{aligned} &\tau_k^h = \max\{\tau^h(t)\} = h_k, \\ &\tau_k^h \in \mathrm{T}^h = \{\mathrm{T}^{h1}, \mathrm{T}^{h2}, \dots, \mathrm{T}^{hl}\}, \quad l \in \mathbb{N} \end{aligned}$$

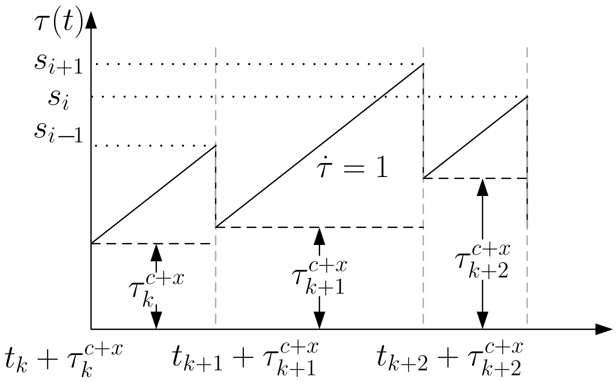

Figure 4 The evolution of time-varying delay $\tau(t)$.

The appearance of τ_k^h is i. i. d., see (5) and [17]. According to the convolution law of independent random variable [22], select s_i in (10) as any subset S

$$s_i \in S \subset \{\mathrm{T}^c + \mathrm{T}^x + \mathrm{T}^h\}\,.$$

Then, s_i is also i. i. d., see also Fig. 4. This implies that the indicator function β_i is a binary i. i. d. process, i. e. has a Bernoulli distribution. As a result,

$$\mathbf{Pr}\{\beta_i = 1\} = p_i\,, \quad \sum_{i=1}^{n} p_i = 1\,. \tag{14}$$

Furthermore, its expected value and variation are given by

$$\mathbb{E}\{\beta_i\} = p_i\,, \quad \mathbb{E}\{(\beta_i - p_i)^2\} = p_i(1 - p_i)\,.$$

In order to derive the occurrence probability p_i from the occurrence probabilities of the delay components, we categorize τ_k^c, τ_k^x and τ_k^h into $U \geq 1$, $V \geq 1$ and $W \geq 1$ intervals with s_u^c, s_v^x and s_w^h satisfying

$$s_{u-1}^c < s_u^c\,, \quad s_u^c > 0\,, \quad u = 1, \ldots, U - 1\,,$$
$$s_{v-1}^x < s_v^x\,, \quad s_v^x > 0\,, \quad v = 1, \ldots, V - 1\,,$$
$$s_{w-1}^h < s_w^h\,, \quad s_w^h > 0\,, \quad w = 1, \ldots, W - 1\,,$$

with s_u^c, s_v^x, s_w^h taking values in the sets T^c, T^x, T^h respectively. Further assume that the occurrence probabilities of the delay intervals are

$$\mathbf{Pr}\{s_{u-1}^c \leq \tau_k^c < s_u^c\} = p_u^c\,, \quad \sum_{u=1}^{U} p_u^c = 1\,,$$

$$\mathbf{Pr}\{s_{v-1}^x \leq \tau_k^x < s_v^x\} = p_v^x\,, \quad \sum_{v=1}^{V} p_v^x = 1\,,$$

$$\mathbf{Pr}\{s_{w-1}^h \leq \tau_k^h < s_w^h\} = p_w^h\,, \quad \sum_{w=1}^{W} p_w^h = 1\,.$$

The delay intervals in (10) and associated occurrence probabilities of indicator functions in (14) become

$$s_i = \sum_{u=1}^{U}\sum_{v=1}^{V}\sum_{w=1}^{W} s_u^c + s_v^x + s_w^h\,,$$

$$p_i = \sum_{u=1}^{U}\sum_{v=1}^{V}\sum_{w=1}^{W} p_u^c\, p_v^x\, p_w^h\,.$$

Remark 2. Random delays are often modeled by a binary i. i.d random process [9; 23] or a finite-state Markov process [11; 13]. The non-binary i. i. d. assumption on the transmission and computation delays has the advantage of less conservatism in the control design. However, an i. i. d. process is unable to represent the often observed mutual dependency of consecutive delays. As a future work, this limitation can be improved by introducing the piecewise-deterministic Markov process [24] into the delay modeling.

Remark 3. Note that the performance improves as the number of delay intervals s_i and also the number of state feedback controllers K_i are increased. However, a large number of delay intervals will result in higher computational complexity. In particular, the dimension of LMIs increases proportionally with the number of delay intervals as shown later. The determination of an optimal number of delay intervals belongs to the future work.

The switching control mechanism discussed above aims at improving the control performance of NCSs with varying feedback delays. Based on the analysis of delay intervals s_i and the associated occurrence probability p_i in this section, the stability analysis and a controller design approach are introduced in the following sections.

4 Stability Analysis

The objective of this section is to derive a mean exponential stability condition for the system in (11). A Lyapunov-Krasovskii approach is selected to analyze the stability of system (11) as it is stochastic and contains time delays. Generally, the stability conditions derived by a Lyapunov-Krasovskii approach can be categorized into two types; delay-independent and a typically less conservative delay-dependent conditions.

In order to derive a delay-dependent condition, the following Newton-Leibnitz formula is considered

$$\int_{t-s_i}^{t} \dot{x}(s)\, ds = x(t) - x(t - s_i)\,.$$

Substituting the Newton-Leibnitz formula into (13) and defining $z^T(t) = [x^T(t)\; \dot{x}^T(t)]$ results in the closed-loop system

$$E\dot{z}(t) = \bar{A}\; z(t) - \sum_{i=1}^{n} \bar{A}_i \int_{t-s_i}^{t} z(s)\, ds\,, \tag{15}$$

where $E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$,

$$\bar{A} = \begin{bmatrix} 0 & I \\ A + \sum_{i=1}^{n} \beta_i BK_i & -I \end{bmatrix}, \; \bar{A}_i = \begin{bmatrix} 0 & 0 \\ 0 & \beta_i BK_i \end{bmatrix}.$$

The system (15) is used for stability analysis and controller synthesis. The stability condition is represented by an easily computable LMI condition as given in detail in the following theorem.

Theorem 1. *The closed-loop system (11) is mean exponentially stable, if there exist symmetric matrices, $Q_i > 0$, $i = 1, \ldots, n$, $P_1 > 0$ and real matrices P_2 and P_3 with*

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix},$$

such that the following LMI is satisfied

$$\begin{bmatrix} \Psi & s_1 P^T & \cdots & s_n P^T \\ * & -s_1 Q_1 & 0 & \vdots \\ \vdots & 0 & \ddots & * \\ * & \cdots & * & -s_n Q_n \end{bmatrix} < 0, \qquad (16)$$

where

$$\Psi = \begin{bmatrix} \Xi_1 & \Xi_2 \\ P_1 - P_2 & -P_3 \end{bmatrix} + \begin{bmatrix} \Xi_1 & \Xi_2 \\ P_1 - P_2 & -P_3 \end{bmatrix}^T + \sum_{i=1}^{n} s_i \begin{bmatrix} 0 & 0 \\ 0 & p_i BK_i \end{bmatrix}^T Q_i \begin{bmatrix} 0 & 0 \\ 0 & p_i BK_i \end{bmatrix},$$

$$\Xi_1 = A^T P_2 + \sum_{i=1}^{n} p_i (BK_i)^T P_2,$$

$$\Xi_2 = A^T P_3 + \sum_{i=1}^{n} p_i (BK_i)^T P_3.$$

Proof. See Appendix A.1.

The LMI stability condition in Theorem 1 can be efficiently solved by computational toolbox for Matlab, e. g. `Yalmip` [25].

Remark 4. The main difference of stability results derived in [11; 13] and Theorem 1 is that Theorem 1 is conditioned by occurrence probabilities of random delays, while the others are determined by transition generator of Markovian delays.

Thus, NCSs with varying feedback delays and switching controller reformulated in Sect. 3 is mean exponentially stable if the LMI (16) in Theorem 1 is satisfied.

5 Controller Design

Solving for the feedback gains K_i, $i = 1, \ldots, n$ in Theorem 1 involves nonlinear terms, e. g. $P_2^T BK_i$ and $P_3^T BK_i$ in (16). These nonlinear terms render the inequality in (16) into a bilinear matrix inequality (BMI) problem, whose solutions are difficult to find as it is non-convex and NP-hard.

However, the nonlinear terms can be eliminated by choosing a special matrix $X = P^{-1}$ such that an LMI formulation is recovered. The controller design algorithm is given in the following theorem.

Theorem 2. *Given positive scalars r_1 and r_2, if there exist symmetric matrices $R_i > 0$, $i = 1, \ldots, n$, and $X_1 > 0$ satisfying*

$$X = \begin{bmatrix} X_1 & 0 \\ -r_1 X_1 & r_2 X_1 \end{bmatrix},$$

such that

$$\begin{bmatrix} \hat{\Psi} & \hat{\Psi}_1^T & \cdots & \hat{\Psi}_n^T \\ * & -s_1 R_1 & 0 & \vdots \\ \vdots & 0 & \ddots & * \\ * & \cdots & * & -s_n R_n \end{bmatrix} < 0, \qquad (17)$$

where

$$\hat{\Psi} = \begin{bmatrix} -r_1 X_1 & r_2 X_1 \\ \Xi_3 & -r_2 X_1 \end{bmatrix} + \begin{bmatrix} -r_1 X_1 & r_2 X_1 \\ \Xi_3 & -r_2 X_1 \end{bmatrix}^T + \sum_{i=1}^{n} s_i R_i,$$

$$\Xi_3 = AX_1 + \sum_{i=1}^{n} p_i BY_i + r_1 X_1,$$

$$\hat{\Psi}_1 = s_1 \bar{A}_1 X = s_1 \begin{bmatrix} 0 & 0 \\ -p_1 r_1 BY_1 & p_1 r_2 BY_1 \end{bmatrix},$$

$$\vdots$$

$$\hat{\Psi}_n = s_n \bar{A}_n X = s_n \begin{bmatrix} 0 & 0 \\ -p_n r_1 BY_n & p_n r_2 BY_n \end{bmatrix},$$

holds, then the NCS is MES with the feedback gain

$$K_i = Y_i X_1^{-1}, \quad i = 1, \ldots, n. \qquad (18)$$

Proof. See Appendix A.2.

Remark 5. The structure of matrix X is chosen based on the requirement $P^{-1} = X$, where $EP = P^T E$. Therefore, X is determined as follows

$$X = \begin{bmatrix} X_1 & 0 \\ X_2 & X_3 \end{bmatrix}, \quad X_1 = X_1^T > 0. \qquad (19)$$

However, by expanding $\hat{\Psi}_i$, $i = 1, \ldots, n$ in (17) it results in terms, e. g. $BK_i X_j$, $i = 1, \ldots, n$ and $j = 1, 2, 3$, which makes the derivation of an LMI formulation not possible. In order to obtain an LMI formulation, one possibility is to set X_2 and X_3 in (19) as $-n_1 X_1$ and $n_2 X_1$, where n_1 and n_2 are positive real numbers.

Although the LMI algorithm can be efficiently solved by the LMI toolbox, the restriction on matrix X introduces conservatism in the controller design. The design algorithm (17) in Theorem 2 might not provide a feasible solution, even if there exists one.

A less conservative approach is to set X back to (19) and solve the BMI (bilinear matrix inequality) directly. However, solving an BMI has the drawback that the feasible feedback gains can only be found strongly depending on the initial conditions. A numerical search regarding any possible initial conditions in unavoidable, e. g. using V-K iteration [10] or cone complementary linearization [26]. The solution of the LMI algorithm in Theorem 2 can be used as an initial condition for solving the BMI. In this case, less conservative feedback gains can be derived.

The proposed analysis and design approaches in Theorem 1 and Theorem 2 are applicable for systems with aperiodic or periodic sampling intervals. In order to explore aperiodic control systems, an event-based grabbing camera is considered in the following experiment.

6 Networked Visual Servoing Experiment

In order to validate the proposed control approach, a networked visual servoing experiment is performed. In the experiment, the tracking of a moving object by a linear motor module is considered using a camera together with a popular pose estimation algorithm as a position measuring sensor. The experimental testbed consists of linear motor module with an object and a controlled linear motor module equipped with a camera, see also Fig. 5 for a visualization. The position measurement is derived by a pose estimation algorithm implemented on a standalone PC and fed back through the communication network. The resulting NCS contains aperiodic sampling intervals, random transmission and computation delays. The two linear motor modules are connected to a host PC running on real-time Linux via a Sensoray S626 I/O card. The control functions are implemented in Matlab/Simulink blocksets. Standalone real-time code is generated directly from Simulink models.

The position of the controlled module $x(t)$ is measured by using a high-speed camera with a resolution of 640×480 pixels. As an approximation to an event-based system, a high image framerate of 400 fps is used. In comparison with an ideal event-based approach, this results in a jitter of 2.5 ms with respect to the sampling rate of 1 kHz of the robot controller. Compared to the time-

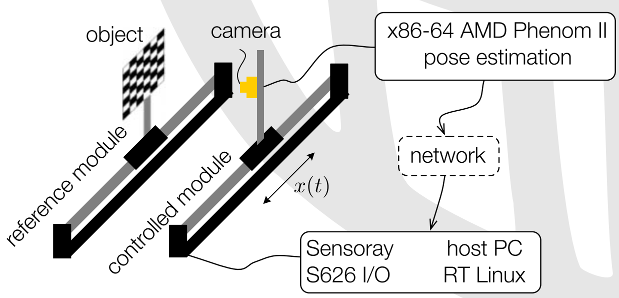

Figure 5 Experimental setup of networked visual servo control.

delays of data transmission and image processing, it can be neglected and the system may thus be approximatively considered as an event-based system. The pose estimation algorithm is implemented on a second PC (x86-64 AMD Phenom II ×4 810 processor). At sampling instant t_k, image processing of the current captured image starts. Sift features [27] are firstly extracted from the image and then matched with the features in the desired image. The matched feature pairs are fed to the pose estimation algorithm [28]. Thus, the position data is determined. The position data is packetized and sent back to the controlled module through the communication network.

It has to be mentioned that the number of matched feature pairs has an impact on the time required for pose estimation. Moreover, image features vary from frame to frame due to different view angles, illumination conditions and noise. Therefore, the position data obtained from the pose estimation are randomly delayed due to varying image features. In addition, a new image is captured as soon as the image processing is finished. This kind of serial image processing mechanism results in aperiodic sampling intervals. As a result, the sampling interval is related to the image processing delay.

The whole computation delay τ_k^c ranges from 33 ms to 45 ms in the experiment. The relationship between image features and image processing delay is shown in Fig. 6, which indicates more image features require longer image processing time. The image feature number has mean value 35.47 and standard deviation 6.27. The position data is fed back to the host PC via a network, which is simulated by Network emulator (`Netem`) having i. i. d. transmission delay τ_k^x ranging from 5 ms to 10 ms. The estimated position $x(t_k)$ arrives at the host PC with random computation delay τ_k^c and transmission delay τ_k^x. As soon as the computation is finished, a new image is required to be processed. Hence, a random sampling interval is resulted. According to (7), the overall feedback delay $\tau(t)$ ranges thus from 38 ms to 100 ms as shown in the Fig. 7a.

Considering $n = 2$ for system in (11), the closed-loop system of the controlled module yields

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.959 & -1169.9 \end{bmatrix}\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + \beta_1(t)\begin{bmatrix} 0 & 0 \\ K_1 & 0 \end{bmatrix}\begin{bmatrix} x(t-s_1) \\ \dot{x}(t-s_1) \end{bmatrix} + \beta_2(t)\begin{bmatrix} 0 & 0 \\ K_2 & 0 \end{bmatrix}\begin{bmatrix} x(t-\bar{\tau}) \\ \dot{x}(t-\bar{\tau}) \end{bmatrix}. \quad (20)$$

The reference module moves along sinusoidal trajectory with the amplitude of 15 cm and frequency of 0.25 Hz. The delay interval $s_1 = 65$ ms is heuristically selected. Parameters $p_1 = 0.53$ and $p_2 = 0.47$ are determined from the experiment. Solving Theorem 2, the feedback gains are

$$K_1 = 817, \quad K_2 = 174.$$

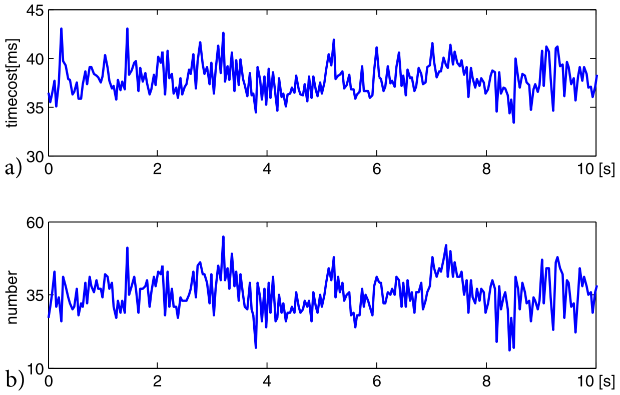

Figure 6 Image processing delay (**a**) and extracted image features (**b**).

The experiment is run 20 times from the same initial condition of both modules. The non-switching controller design, i. e. controller design by $\bar{\tau} = 100$ ms and $K = 174$, and proposed delay-dependent switching controller design approach are compared. The control error is defined by

$$\bar{e}(t) = x_r(t) - x_c(t)\,,$$

where $x_r(t)$ denotes the position of the reference module, and $x_c(t)$ denotes the position of the controlled module. The evolution of mean control error are shown in Fig. 7b. The proposed delay-dependent switching controller has maximal tracking error $\bar{e}_{\max} = 2.89$ cm and variance of tracking error $\bar{e}_{\text{var}} = 224\ \text{cm}^2$, while the maximal tracking error of non-switching design controller is $\bar{e}_{\max} = 9.38$ cm and the variance $\bar{e}_{\text{var}} = 620\ \text{cm}^2$.

The experimental results show that controller design algorithm in Theorem 2 enables a good control performance compared to the conventional non-switching design.

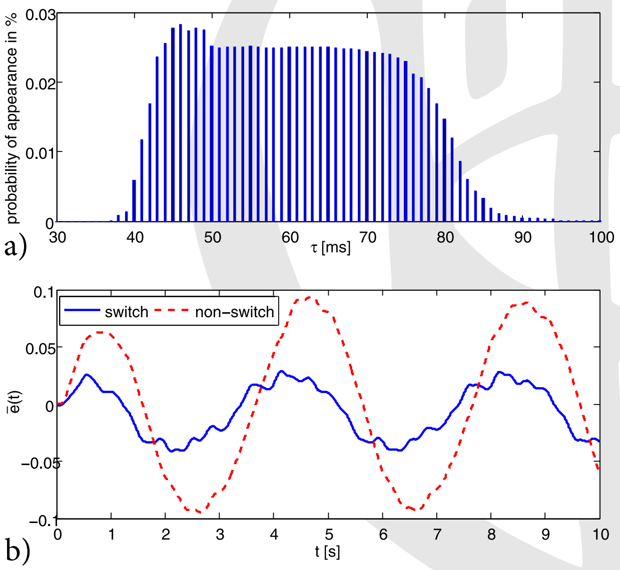

Figure 7 The delay histogram of feedback signals (**a**); and mean control error evolution of delay-dependent controller (solid line) non-switching design controller (dash line) (**b**).

7 Conclusion

This article presents a novel analysis and design approach for networked control systems (NCSs) with aperiodic sampling intervals as well as random transmission and computation delays using delay-dependent switching controllers. By applying an input-delay approach to the NCS, the transmission delay, computation delay and the aperiodic updates at the receiver are reformulated into a random compound delay. The compound delay is divided into n intervals. The statistical properties of n delay intervals determine the stability certificate in terms of mean exponential stability. Also, the switching delay-dependent controllers are determined by the occurrence probabilities of the n delay intervals. The proposed approach is validated in a networked visual servoing experiment. Experimental results demonstrate the superior performance of the proposed design approach over the conventional non-switching counterpart.

A Appendix

Before the proof is shown, the following definition and lemma have to be given.

Definition 2. *[29] Let $\mathcal{L}$ be the infinitesimal generator of a function $V(z(t))$. Then, the operator $\mathcal{L}$ acting on $V(z(t))$ is defined as*

$$\mathcal{L}V(z(t)) = \lim_{\Delta \to 0} \frac{1}{\Delta}\Big\{\mathbb{E}\{V(z(t+\Delta)|z(t))\} - V(z(t))\Big\}\,.$$

Lemma 1. *[30] Let X and Y be real constant matrices with appropriate dimensions. Then*

$$X^T Y + Y^T X \le \varepsilon X^T X + \frac{1}{\varepsilon} Y^T Y$$

holds for any $\varepsilon > 0$.

A.1 Proof of Theorem 1

Consider a Lyapunov-Krasovskii functional candidate

$$V(z(t)) = V_0(z(t)) + \sum_{i=1}^{n} V_i(z(t))\,,$$

where

$$V_0(z(t)) = z^T(t) E P z(t)\,,$$

$$V_i(z(t)) = \int_{-s_i}^{0}\int_{t+\theta}^{t} z^T(s)\bar{A}_i^T Q_i \bar{A}_i z(s)\, ds\, d\theta\,.$$

According to Definition 1, it has

$$\begin{aligned}\mathcal{L}V_0(z(t)) &= \dot{z}^T(t) E P z(t) + z^T(t) P^T E \dot{z}(t)\\ &= z^T(t)[\bar{A}^T P + P^T \bar{A}] z(t)\\ &\quad - 2\sum_{i=1}^{n} z^T(t) P^T \bar{A}_i \int_{t-s_i}^{t} z(s)\, ds\,.\end{aligned}$$

According to Lemma 1, $\mathcal{L}V_0(z_t)$ becomes

$$\mathcal{L}V_0(z(t)) \le z^T(t)[\bar{A}^TP + P^T\bar{A}]z(t) + \sum_{i=1}^{n} s_i z^T(t)P^TQ_i^{-1}Pz(t) + \sum_{i=1}^{n}\int_{t-s_i}^{t} z^T(s)\bar{A}_i^TQ_i\bar{A}_iz(s)\,ds\,. \quad (A.1)$$

Likewise, it has

$$\sum_{i=1}^{n}\mathcal{L}V_i(z(t)) = \sum_{i=1}^{n} s_i z^T(t)\bar{A}_i^TQ_i\bar{A}_iz(t) - \sum_{i=1}^{n}\int_{t-s_i}^{t} z^T(s)\bar{A}_i^TQ_i\bar{A}_iz(s)\,ds\,. \quad (A.2)$$

Combine (A.1) and (A.2), it yields

$$\mathcal{L}V(z(t)) \le z^T(t)[\bar{A}^TP + P^T\bar{A} + \sum_{i=1}^{n} s_i\bar{A}_i^TQ_i\bar{A}_i + \sum_{i=1}^{n} s_iP^TQ_i^{-1}P]z(t) = z^T(t)\Theta z(t)\,. \quad (A.3)$$

Apply Schur complement to (A.3), it results in (16).

Note that $\max_{\theta\in[-\tau,0]}\{||z(t+\theta)||\} \le \phi||z(t)||$ for some $\varphi > 0$ [31], the following inequality can be established

$$V(z(t)) \le \left[\lambda_{\max}(EP) + \phi\sum_{i=1}^{n}\frac{s_i^2}{2}\lambda_{\max}(\bar{A}_i^TQ_i\bar{A}_i)\right]||z(t)||^2 \le \Lambda_{\max}||z(t)||^2\,. \quad (A.4)$$

Combining (A.3) and (A.4) yields

$$\frac{\mathcal{L}V(z(t))}{V(z(t))} \le -\frac{\lambda_{\min}(-\Theta)}{\Lambda_{\max}} \triangleq -\rho_0$$

and

$$\mathbb{E}\{\mathcal{L}V(z(t))\} \le -\rho_0\mathbb{E}\{V(z(t))\}\,. \quad (A.5)$$

By applying Dynkin's formula into (A.5) it becomes

$$\mathbb{E}\{V(z(t))\} - \mathbb{E}\{V(z(0))\} = \mathbb{E}\left\{\int_0^t \mathcal{L}V(z(s))ds\right\} \le -\rho_0\int_0^t \mathbb{E}\{V(z(s))\}\,ds\,. \quad (A.6)$$

Using the Gronwall-Bellman lemma, (A.6) results in

$$\mathbb{E}\{V(z(t))\} \le e^{-\rho_0 t}\mathbb{E}\{V(z(0))\}\,.$$

Since

$$V(z(t)) \ge \left[\lambda_{\min}(EP) + \sum_{i=1}^{n}\frac{s_i^2}{2}\lambda_{\min}(Q_i)\right]||z(t)||^2 = \Lambda_{\min}||z(t)||^2\,,$$

it is established that

$$\mathbb{E}\{||z(t)||^2\} \le e^{-\rho_0 t}\frac{\mathbb{E}\{V(z(0))\}}{\Lambda_{\min}}\,. \quad (A.7)$$

Equation (A.7) provides the proof for mean exponential stability. □

A.2 Proof of Theorem 2

Define

$$X = P^{-1} = \begin{bmatrix} X_1 & 0 \\ -r_1X_1 & r_2X_1 \end{bmatrix}.$$

Pre- and post-multiply Θ in (A.3) by X^T and X, it becomes

$$\bar{A}X + X^T\bar{A}^T + \sum_{i=1}^{n} s_iQ_i^{-1} + \sum_{i=1}^{n} s_iX^T\bar{A}_i^TQ_i\bar{A}_iX < 0 \quad (A.8)$$

Let $R_i = Q_i^{-1}$ and $Y_i = K_iX_1$, $i = 1,\dots,n$. Applying Schur complement to (A.8) results in (17). □

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